Appendices

A Downward-sloping demand for satellites

In this Appendix, we extend the simple model to the case of per-period returns that depends negatively on the stock of satellites in orbit. Our goal is to show that Kessler Syndrome is still possible, with the conditions for its emergence appropriately modified. We begin by considering a technology that uses satellites to produce output. This output is an aggregate bundle of goods and services provided by different types of satellites, e.g. a composite good incorporating telecommunications, imaging, etc. We normalize the price of the composite output to 1, and the unitary cost of a satellite input as $\pi > 0$. The representative aggregator firm takes the unitary cost of a satellite as given and maximizes the per-period profits

$$\pi Z_t^{1+\eta} - p_t Z_t.$$

where Z_t denote the number of satellites *operating* at time t. The solution to the maximization problem corresponds to the demand faced by the satellite operators:

$$\pi(1+\eta)Z_t^{\eta} = p_t.$$

The case considered in the main text is equivalent to a situation where the aggregation technology is constant returns to scale, so that $\eta = 0$, and $p_t = \pi$. The unitary return of a satellite can still change over time, something that we do assume in the fully dynamic model, but the change is exogenous to the stock of satellites, and it corresponds to a time-varying productivity π.

Let us consider the case of $\eta < 0$, which implies that the more satellites operating in orbit, the lower the unitary return. An immediate implication of this negative relationship is that as the orbit fills with satellites, the return on satellites declines, and so does the incentive to launch additional satellites. It may appear that a downward-sloping demand curve makes it more difficult to congest the orbit and obtain Kessler Syndrome. However, this argument is incomplete. It is important to recognize that the orbit congestion depends on the total number of objects in orbit, while the unitary return depends on the objects in orbit that are *still* operating. In other words, as the number of objects in orbit increases, the unitary return might also be increasing if the increase in the number of orbiting objects is primarily due to the increase in debris while operating satellites decline!

To see this in the context of our simple model, consider that in period $t = s$ the number of operating satellites is

$$Z = q(S)S \tag{26}$$

while the total number of objects in orbit—which matters for the survival probability—is the number of satellites launched at $t = 0$, S. So the unitary return in period $t = s$ decreases as S increases only if

$$\frac{dZ}{dS} = q'(S)S + q(S) \geq 0. \tag{27}$$

If this condition does not hold, as more satellites are launched, the impact on the survival probability dominates the impact on the operating satellites, and the per-period returns are actually increasing in S, even when there is a downward-sloping demand for satellites. If we let $q(X) = 1 - S/\bar{X}$, the condition above is violated whenever $S > \frac{1}{2}\bar{X}$, that is whenever the orbit is sufficiently congested. The insight offered by this simple example is that a downward-sloping demand for satellites, when combined with collision risk, might end up exacerbating the incentive to congest the orbit. We believe this positive feedback mechanism is interesting, but we leave a systematic analysis of its implications for future work. For the purpose of the current analysis we consider the argument above as a reassurance that a constant return π is not an unreasonable assumption for our baseline model.

Assuming that operators face the demand curve above, one can show that the condition for Kessler Syndrome under open access becomes

$$F \leq \frac{\pi(1+\eta)}{1+r} S_K^{\eta} q(S_K)^{1+\eta}. \tag{28}$$

Compared to the case of $\eta = 0$, Kessler Syndrome is more likely to occur in the presence of a downward-sloping demand when

$$\left[S_K q(S_K)\right]^{\eta} > \frac{1}{1+\eta}. \tag{29}$$

Using once again the functional form $q(X) = 1 - S/\bar{X}$, this condition corresponds to

$$\left[\bar{X}\left(\frac{\sqrt{1+4\sigma}-1}{2\sigma}\right)\left(\frac{1+2\sigma-\sqrt{1+4\sigma}}{2\sigma}\right)\right]^{\eta} > \frac{1}{1+\eta}. \tag{30}$$

Numerical computations show that the impact of η is non-monotonic. Setting $\bar{X} = 1$ and $\sigma = 1$ the inequality above holds approximately for $\eta \in (-0.5, 0)$, with a peak in the gap at around -0.3, which means that for a moderately downward-sloping demand, the positive feedback mechanism described above is strongest.

Taken together, the results just presented indicate that the introduction of a downward-sloping demand has an ambiguous effect on the emergence of Kessler Syndrome. Our maintained assumption of a constant return π corresponds to balancing the two contrasting effects highlighted above. The positive effect articulated here is an interesting and potentially important extension of our analysis that we leave to future work.

B Proofs and derivations

B.1 Proofs omitted from main text

Proposition 1 (Kessler Syndrome)**.** *Let the dynamic model of objects in orbit be characterized by equation (1) with $\sigma > 0$, and the Kessler threshold S_K be defined as in equation (5), then*

1. *under the open-access equilibrium, Kessler Syndrome occurs if*

$$\frac{\pi}{1+r} q(S_K) \geq F; \tag{6}$$

2. *under the social planner allocation, Kessler Syndrome occurs if*

$$\frac{\pi}{1+r}\left[q(S_K) + S_K q'(S_K)\right] \geq F. \tag{7}$$

Proof. First, recall the definition of Kessler Syndrome in the simple model: a launch rate S such that $S < \bar{X}$ while $g(S) \geq \bar{X}$. The smallest level of S at which this condition can hold is S_K, since

$g(S_K) = \bar{X}$, $g(S)$ is increasing in S when $\sigma > 0$, and by assumption $S_K < \bar{X}$.

Suppose open access will cause Kessler Syndrome. Then $\hat{S}$ must be such that $\hat{S} < \bar{X}$ while $g(\hat{S}) \geq \bar{X}$, implying $\hat{S} \geq S_K$. The equilibrium condition becomes

$$F = q(\hat{S})\frac{\pi}{1+r}. \tag{31}$$

Since q is decreasing in S, the above condition can be satisfied if and only if

$$F \leq q(S_K)\frac{\pi}{1+r}. \tag{32}$$

Next, suppose the social planner will cause Kessler Syndrome. Then S^* must be such that $S^* < \bar{X}$ while $g(S^*) \geq \bar{X}$, implying $S^* \geq S_K$. The optimality condition becomes

$$F = [q(S^*) + S^* q'(S^*)]\frac{\pi}{1+r}. \tag{33}$$

Since q is decreasing in S, the above condition can be satisfied if and only if

$$F \leq [q(S_K) + S^* q'(S_K)]\frac{\pi}{1+r}. \tag{34}$$

This completes the proof. □

Proposition 2 (Multiplicity and instability)**.** *Given a positive excess return on a satellite and a collision probability function which depends on satellites and debris, multiple open-access steady states can exist if debris objects can collide and produce new debris ($G_D > 0$). An open-access steady state will be stable if and only if*

$$\underbrace{(G_D(S^*, D^*) - \delta)}_{\substack{\textit{Net rate of} \\ \textit{autocatalytic debris growth}}} < \underbrace{\frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(\frac{\pi}{F} - r))}_{\substack{\textit{Rate of new fragment reduction due to} \\ \textit{equilibrium launch activity response to debris}}}. \tag{21}$$

When G is strictly convex in both arguments and two steady states exist, the higher-debris steady state is unstable.

Proof. The proposition asserts:

1. *Existence of multiple steady states:* Given a positive excess return on a satellite, multiple open-access steady states can exist if debris objects can collide and produce new debris.

2. *Stability of steady states:* An open-access steady state will be stable if and only if

$$(G_D(S^*, D^*) - \delta) < \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(\frac{\pi}{F} - r)). \quad ((21))$$

3. *Ordering of steady states:* When G is strictly convex in both arguments and two steady states exist, the higher-debris is unstable.

Before proving them, we establish a useful reduction.

0. A useful reduction: The open-access steady states are defined by equations (8), (9), and (15), combined with the conditions $D_t = D_{t+1} = D$ and $S_t = S_{t+1} = S$. Since L is monotone increasing in both arguments it is invertible, and equation (15) implicitly determines the number of satellites as a function of the amount of debris, the excess return on a satellite, and the collision rate function,

$$L(S, D) = \frac{\pi}{F} - r \implies S = S(\frac{\pi}{F} - r, D). \quad (35)$$

Since L is monotone increasing in each argument, $S(\frac{\pi}{F} - r, D)$ is monotone decreasing in D. Since S must be nonnegative, there exists a nonnegative $D^S : S(\frac{\pi}{F} - r, D) = 0\ \forall D \geq D^S$. Let $\hat{S}$ be the equilibrium satellite stock as a function of the debris stock. So we have

$$\hat{S} = \begin{cases} S(\frac{\pi}{F} - r, D) \text{ if } D \in [0, D^S) \\ 0 \text{ if } D \geq D^S \end{cases} \quad (36)$$

Using $\hat{S}$ we can reduce equations (8), (9), and (15) to a single equation in debris,

$$\mathscr{Y}(D) = -\delta D + G(\hat{S}, D) + m(\frac{\pi}{F} - r)\hat{S},$$

with the solutions

$$\{\hat{D} \geq 0 : \delta \hat{D} = G(\hat{S}, \hat{D}) + m(\frac{\pi}{F} - r)\hat{S}\} \tag{37}$$

being the open-access steady states.

1. Existence of multiple steady states: Using the above reduction, we focus our attention on solutions to equation (B.1). δD is monotonically increasing in D with $\delta D = 0$ when $D = 0$, and $m(\frac{\pi}{F} - r)\hat{S}$ is monotonically decreasing in D with $\hat{S} > 0$ when $D = 0$, but $\hat{G} \equiv G(\hat{S}, D)$ is nonmonotone in D. To see this, note

$$\frac{d\hat{G}}{dD}(\hat{S}, D) = \underbrace{\frac{\partial G}{\partial S}}_{\geq 0} \underbrace{\frac{\partial \hat{S}}{\partial D}}_{\leq 0} + \underbrace{\frac{\partial G}{\partial D}}_{\geq 0}, \text{ with} \tag{38}$$

$$\frac{d\hat{G}}{dD}(\hat{S}, 0) = \frac{\partial G}{\partial S}\frac{\partial \hat{S}}{\partial D} < 0 \text{ and} \tag{39}$$

$$\frac{d\hat{G}}{dD}(0, D^S) = \frac{\partial G}{\partial D} > 0, \tag{40}$$

where $\frac{\partial \hat{S}}{\partial D} = -\frac{L_D}{L_S} \leq 0$ by application of the Implicit Function Theorem on equation (15).

Let $\hat{D}$ be a solution to equation (B.1). If $G_D > 0$, then $\hat{G}$ is nonmonotone in D and the existence or uniqueness of $\hat{D}$ cannot be guaranteed. If G_D is large enough, $\hat{D}$ will not exist; if G_D is not too small, multiple $\hat{D}$ will exist. If $G_D = 0$, then the existence of $\hat{D}$ also ensures its uniqueness. If G_D is strictly convex in both arguments, at most two $\hat{D}$ can exist.

2. Stability of steady states: Since $\mathscr{Y}(D)$ is a reduction of the open-access dynamical system, its fixed points are isomorphic to the fixed points of equations (8), (9), and (15). The sign of $\frac{\partial \mathscr{Y}}{\partial D}$ at solutions to $\mathscr{Y}(D) = 0$ matches the sign of the respective eigenvalues of the full system.

Applying the Implicit Function Theorem to equation (35) to calculate S_D and then differentiating $\mathscr{Y}$ in the neighborhood of an arbitrary solution D^*, we obtain

$$\frac{\partial \mathscr{Y}}{\partial D}(D^*) = (G_D(S^*, D^*) - \delta) - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(\frac{\pi}{F} - r)), \tag{41}$$

where $S^* \equiv S(\frac{\pi}{F} - r, D^*)$. Both $G_S(S^*, D^*)$ and $m(\frac{\pi}{F} - r)$ are positive by assumption. So $\frac{\partial \mathscr{Y}}{\partial D}(D^*) < 0$ holds if and only if δ is small enough, or $\frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}$ is large enough, i.e.

3. Ordering of steady states: When G is strictly convex in both arguments and $\mathscr{Y}(D) = 0$ has two solutions. Denote the smaller solution by $\underline{D}$, and the larger solution by $\bar{D}$. The curve $G(\hat{S}, D) + m(\frac{\pi}{F} - r)\hat{S}$ is above δD when $D = 0$, and again as $D \to \infty$. $\mathscr{Y}(D)$ must therefore approach 0 from above as $D \to \underline{D}$ from the left, and from below as $D \to \bar{D}$ from the left. This implies that at $\underline{D}$,

$$\frac{\partial \mathscr{Y}}{\partial D}(\underline{D}) = (G_D(\underline{S}, \underline{D}) - \delta) - \frac{L_D(\underline{S}, \underline{D})}{L_S(\underline{S}, \underline{D})}(G_S(\underline{S}, \underline{D}) + m(\frac{\pi}{F} - r)) < 0 \tag{42}$$

and at the second solution, $\bar{D}$,

$$\frac{\partial \mathscr{Y}}{\partial D}(\bar{D}) = (G_D(\bar{S}, \bar{D}) - \delta) - \frac{L_D(\bar{S}, \bar{D})}{L_S(\bar{S}, \bar{D})}(G_S(\bar{S}, \bar{D}) + m(\frac{\pi}{F} - r)) > 0. \tag{43}$$

where $\underline{S} = \hat{S}(\underline{D})$ and $\bar{S} = \hat{S}(\bar{D})$. □

Proposition 3 (Overshooting)**.** *Suppose the new fragment formation function is strictly convex in both arguments and the launch rate constraint does not bind. Except on a set of measure zero, open access paths from initial conditions with positive launch rates will overshoot the stable open-access steady state in at least one state variable.*

Proof. We first define the following sets and functions, where $S, D \geq 0$ is assumed:

- The action region: the set of states with positive open-access launch rates,

$$A \equiv \left\{(S, D) : \frac{\pi}{F} - r - L(S', D') \geq 0\right\}, \tag{44}$$

where

$$S' = S(1 - L(S, D)) + X$$
$$D' = D(1 - \delta) + G(S, D) + mX,$$

- The equilibrium manifold:

$$E \equiv \left\{(S, D) : \frac{\pi}{F} - r - L(S, D) = 0\right\}. \tag{45}$$

- The stable open-access steady state: following the reduction used in the proof of Proposition 2, we characterize the stable open-access steady state as

$$E_s \equiv \Big\{ (\hat{S}, D) : \mathscr{Y}(D) = -\delta D + G(\hat{S}, D) + m(\frac{\pi}{F} - r)\hat{S} = 0, \\ \hat{S} : L(\hat{S}, D) = \frac{\pi}{F} - r, \quad \mathscr{Y}'(D) < 0 \Big\}. \tag{46}$$

- The physical dynamics: the mapping $P_{SD} : \mathbb{R}^2_+ \to \mathbb{R}^2_+$ which describes the effect of orbital mechanics on the satellite and debris stocks in one period,

$$P_{SD}(S, D) \equiv \big(S(1 - L(S, D)), \quad D(1 - \delta) + G(S, D)\big). \tag{47}$$

- The one-step set: the set of states from which one period's physical dynamics, followed by launching, will read an open-access steady state,

$$A_{P1} \equiv \{(S, D) : P_{SD}(S, D) + (X, mX) \in E_s, \quad X \in (0, \bar{X}]\}. \tag{48}$$

- The one-step ray: the set of states from which one period of launching will reach an open-access steady state,

$$A_1 \equiv \big\{(S, D) : (S + X, D + mX) \in E_s, \quad X \in (0, \bar{X}]\big\}, \tag{49}$$

  where m is the same as in the debris law of motion. The one-step ray can be viewed as part of a decomposition of the satellite and debris laws of motion: after a period's physical dynamics have been applied, launches to the stable steady state occur from the one-step ray. The one-step set encompasses both of these components.

Our proof proceeds in three steps. First, we show that initial conditions in the action region A reaching points on the equilibrium manifold $E \setminus E_S$ must overshoot an open-access steady state. Second, we establish the bijectivity of the physical dynamics P_{SD}. Third, we show that these results imply that the one-step set A_{P1} has zero Lebesgue measure on A.

1. Initial conditions in the action region A reaching points on the equilibrium man-

ifold $E \setminus E_S$ must overshoot an open-access steady state: Since the launch rate constraint does not bind, any point in A will by definition reach a point in E. Since E_s contains at most one element given the strict convexity of G while E is a manifold, $E_s \subset E$. Given that L is increasing in both arguments, points in $E \setminus E_s$ must therefore have either larger S and smaller D than E_s, or vice versa. Consequently, reaching points in $E \setminus E_s$ constitutes overshooting E_s in one state variable and undershooting in the other.

2. Bijectivity of the physical dynamics P_{SD}: To show that P_{SD} is a bijection on $\mathbb{R}^2_+$, we separate the physical dynamics into two functions $P_S, P_D : \mathbb{R}^2_+ \to \mathbb{R}_+$,

$$P_S(S, D) = S(1 - L(S, D)), \tag{50}$$

$$P_D(S, D) = D(1 - \delta) + G(S, D). \tag{51}$$

P_D is a sum of strictly monotone increasing functions, so is strictly monotone increasing as well. Strictly monotone functions are bijections, so P_D is a bijection. So for two arbitrary pairs (S_1, D_1) and (S_2, D_2) we have

$$P_{SD}(S_1, D_1) = P_{SD}(S_2, D_2) \iff P_S(S_1, D_1) = P_S(S_2, D_2) \ \& \ P_D(S_1, D_1) = P_D(S_2, D_2) \tag{52}$$

P_S is a function, so we have $(S_1, D_1) = (S_2, D_2) \implies P_S(S_1, D_1) = P_S(S_2, D_2)$, but since $SL(S, D)$ may be non-monotone the other direction may not hold. Since P_D is a bijection, $P_D(S_1, D_1) = P_D(S_2, D_2) \iff (S_1, D_1) = (S_2, D_2)$. Putting this together we have the following:

- If $(S_1, D_1) = (S_2, D_2)$, then $P_{SD}(S_1, D_1) = P_{SD}(S_2, D_2)$.
- If $P_{SD}(S_1, D_1) = P_{SD}(S_2, D_2)$, then $P_S(S_1, D_1) = P_S(S_2, D_2)$ and $P_D(S_1, D_1) = P_D(S_2, D_2)$. While there may exist a pair $(S_1, D_1) \neq (S_2, D_2)$ such that $P_S(S_1, D_1) = P_S(S_2, D_2)$, the bijectivity of P_D means $P_D(S_1, D_1) \neq P_D(S_2, D_2)$.

Consequently, $P_{SD}(S_1, D_1) = P_{SD}(S_2, D_2)$ if and only if $(S_1, D_1) = (S_2, D_2)$, i.e. $P_S D$ is a bijection on $\mathbb{R}^2_+$.

3. The one-step set A_{P1} has zero Lebesgue measure on A:

By definition, $A_1 \subseteq A$. Since E_s contains at most one element, A_1 is a single line segment, so $A_1 \subset A$. The Lebesgue measure on A of A_1 is therefore zero.

Since P_{SD} is a bijection, the Lebesgue measure of the pre-image of A_1 under P_{SD},

$$P_{SD}^{-1}(A_1) \equiv \left\{(S, D) : P_{SD}(S, D) \in A_1\right\},$$

is the same as the Lebesgue measure of A_1. Since $P_{SD}^{-1}(A_1) = A_{P1}$, the Lebesgue measure on A of A_{P1} is also zero. Lebesgue measure is isomorphic to any non-atomic probability measure, so the one-step set is measure zero under any non-atomic probability measure. This gives the desired result: initial conditions with positive open-access launch rates will overshoot the stable open-access steady state except on a set of measure zero.

□

B.2 Optimal launch policy and external cost

The infinite-horizon sequence version of the fleet planner's problem is

$$\max_{\{X_t, S_{t+1}, D_{t+1}\}_{t=0}^{\infty}} S_t Q(S_t, D_t, X_t) + \frac{1}{1+r} \sum_{\tau=t}^{\infty} \frac{1}{1+r}^{\tau-t-1} X_\tau \left(\frac{1}{1+r} Q(S_{\tau+1}, D_{\tau+1}, X_{\tau+1}) - F\right) \quad (53)$$

$$\text{s.t. } Q(S_t, D_t, X_t) = \pi + \frac{1}{1+r}(1 - L(S_t, D_t)) Q(S_{t+1}, D_{t+1}, X_{t+1}) \quad (54)$$

$$S_{t+1} \leq S_t(1 - L(S_t, D_t)) + X_t \quad (55)$$

$$D_{t+1} \geq D_t(1 - \delta) + G(S_t, D_t) + m X_t \quad (56)$$

$$X_t \in [0, \bar{X}] \ \ \forall t \quad (57)$$

$$S_{t+1} \geq 0, D_{t+1} \geq 0 \quad (58)$$

$$S_0 = s_0, D_0 = d_0 \quad (59)$$

For generality, we include an upper bound $\bar{X}$ on the allowable launch rate. If this never binds then the appropriate shadow value will simply be identically zero ($\gamma_{\bar{X}_t} \equiv 0$). The planner's Lagrangian is

$$\mathscr{L}(X,S,D,\lambda,\gamma) = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \Big\{\pi S_t - FX_t + \lambda_{S_t}\left(S_t(1-L(S_t,D_t)) + X_t - S_{t+1}\right)$$

$$+ \lambda_{D_t}\left(D_{t+1} - D_t(1-\delta) - G(S_t,D_t) - mX_t\right)$$

$$+ \gamma_{X_t}X_t + \gamma_{\bar{X}_t}(\bar{X} - X_t) + \gamma_{S_t}S_{t+1} + \gamma_{D_t}D_{t+1}\Big\} \tag{60}$$

The first-order necessary conditions for an optimal launch path are, $\forall t$ up to T,

$$\mathscr{L}_{X_t} = -F + \lambda_{S_t} - m\lambda_{D_t} + \gamma_{X_t} - \gamma_{\bar{X}_t} = 0 \tag{61}$$

$$\mathscr{L}_{S_{t+1}} = \frac{1}{1+r}\{\pi + \lambda_{S_{t+1}}(1 - L(S_{t+1},D_{t+1}) - S_{t+1}L_S(S_{t+1},D_{t+1}))$$

$$- \lambda_{D_{t+1}}G_S(S_{t+1},D_{t+1})\} + \gamma_{S_t} - \lambda_{S_t} = 0 \tag{62}$$

$$\mathscr{L}_{D_{t+1}} = \frac{1}{1+r}\{\lambda_{D_{t+1}}(\delta - 1 - G_D(S_{t+1},D_{t+1})) - \lambda_{S_{t+1}}S_{t+1}L_D(S_{t+1},D_{t+1})\} + \lambda_{D_t} + \gamma_{D_t} = 0 \tag{63}$$

$$\mathscr{L}_{S_{T+1}} = \gamma_{S_T} - \lambda_{S_T} = 0 \tag{64}$$

$$\mathscr{L}_{D_{T+1}} = \lambda_{D_T} + \gamma_{D_T} = 0 \tag{65}$$

with complementary slackness and transversality conditions

$$\lambda_{St}\left(S_t(1 - L_t + X_t - S_{t+1}\right) = 0 \tag{66}$$

$$\lambda_{Dt}\left(D_{t+1} - D_t(1-\delta) - G_t - mX_t\right) = 0 \tag{67}$$

$$\gamma_{Xt}X_t = 0, \tag{68}$$

$$\gamma_{\bar{X}t}(\bar{X} - X_t) = 0, \tag{69}$$

$$\gamma_{St}S_{t+1} = 0, \tag{70}$$

$$\gamma_{Dt}D_{t+1} = 0 \tag{71}$$

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T \lambda_{ST}S_{T+1} = 0 \tag{72}$$

$$\lim_{T\to\infty} -\left(\frac{1}{1+r}\right)^T \lambda_{DT}D_{T+1} = 0. \tag{73}$$

In what follows we drop time subscripts to reduce notational clutter. Period t values are shown with no subscript, period $t+1$ values are marked with a $'$ after the variable, and period $t-1$ values are marked with a $'$ before the variable e.g. $S_{t-1} \equiv {}'S$, $S_t \equiv S$, $S_{t+1} \equiv S'$. By (61),

$$\lambda_S = (1+r)(F + \frac{1}{1+r} m\lambda_D - \gamma_X + \gamma_{\bar{X}}). \tag{74}$$

In the next period, this becomes

$$\lambda'_S = (1+r)(F + \left(\frac{1}{1+r}\right) m\lambda'_D - \gamma'_X + \gamma'_{\bar{X}}). \tag{75}$$

By (62) and (63),

$$\lambda_S = \pi + (1+r)\gamma_S + \frac{1}{1+r}\{\lambda'_S(1 - L(S', D') - S'L_S(S', D')) - \lambda'_D G_S(S', D')\} \tag{76}$$

$$\lambda_D = \frac{1}{1+r}\{\lambda'_D(1 + G_D(S', D') - \delta) + \lambda'_S S' L_D(S', D')\} - (1+r)\gamma_D. \tag{77}$$

Using (75),

$$\lambda_S = \pi + (1+r)\gamma_S - F(L(S', D') + S'L_S(S', D') - 1) - \frac{1}{1+r}\lambda_D G_S(S', D') \tag{78}$$

$$- \frac{1}{1+r} m\lambda_D (L(S', D') + S'L_S(S', D') - 1) + (L(S', D') + S'L_S(S', D') - 1)(\gamma'_X - \gamma'_{\bar{X}}) \tag{79}$$

$$\lambda_D = FS'L_D(S', D') + \frac{1}{1+r}\lambda'_D(1 + G_D(S', D') - \delta) + \frac{1}{1+r} m\lambda_D S' L_D(S', D') \tag{80}$$

$$- \left((1+r)\gamma_D + S'L_D(S', D')(\gamma'_X - \gamma'_{\bar{X}}\right) \tag{81}$$

Define

$$\alpha_1' = \pi + (1 - L(S', D') - S' L_S(S', D'))F \tag{82}$$

$$\alpha_2' = S' L_D(S', D')F \tag{83}$$

$$\Gamma_1' = G_S(S', D') - m(1 - L(S', D') - S' L_S(S', D')) \tag{84}$$

$$\Gamma_2' = 1 - \delta + G_D(S', D') + mS' L_D(S', D') \tag{85}$$

$$\kappa_1' = (1 + r)\gamma_S - (\gamma_{X'} - \gamma_{\bar{X}'})(1 - L(S', D') - S' L_S(S', D')) \tag{86}$$

$$\kappa_2' = (1 + r)\gamma_D + S' L_D(S', D')(\gamma_{X'} - \gamma_{\bar{X}'}), \tag{87}$$

so that

$$\lambda_S = \alpha_1' - \frac{1}{1+r}\lambda_D'\Gamma_1' + \kappa_1' \tag{88}$$

$$\lambda_D = \alpha_2' + \frac{1}{1+r}\lambda_S'\Gamma_2' - \kappa_2'. \tag{89}$$

Then,

$$\lambda_D' = \frac{\lambda_D - \alpha_2' + \kappa_2'}{\frac{1}{1+r}\Gamma_2'}. \tag{90}$$

Substitute (74) and (90) in (88) to get the following expression for $W_D(S, D)$

$$\frac{\frac{1}{1+r}\{\Gamma_1'(\alpha_2' - \kappa_2') + \Gamma_2'(\alpha_1' + \kappa_1')\} + \Gamma_2'(\gamma_X - \gamma_{\bar{X}} - F)}{\frac{1}{1+r}(\Gamma_1' + m\Gamma_2')}. \tag{91}$$

Iterate 91 to period $t + 1$ and substitute into 90 to obtain

$$\lambda_D' = \frac{\frac{1}{1+r}\{\Gamma_1''(\alpha_2'' - \kappa_2'') + \Gamma_2''(\alpha_1'' + \kappa_1'')\} + \Gamma_2''(\gamma_X' - \gamma_{\bar{X}}' - F)}{\frac{1}{1+r}(\Gamma_1'' + m\Gamma_2'')}. \tag{92}$$

Use (91) and (92) in (89) to get

$$\alpha_1' = m(\alpha_2' - \kappa_2') - \kappa_1' + \frac{1}{\frac{1}{1+r}}(\gamma_{\bar{X}} - \gamma_X + F) + \frac{\Gamma_1' + m\Gamma_2'}{\Gamma_1'' + m\Gamma_2''}\left(\Gamma_1''\frac{1}{1+r}(\alpha_2'' - \kappa_2'') + \Gamma_2''(\frac{1}{1+r}(\alpha_1'' + \kappa_1'') - F + \gamma_X' - \gamma_{\bar{X}}')\right). \tag{93}$$

Evaluate (93) in the previous time period as:

$$\alpha_1 = m(\alpha_2-\kappa_2)-\kappa_1+\frac{1}{\frac{1}{1+r}}({}'\gamma_{\bar{X}} -{}' \gamma_X + F)+\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}\left(\Gamma_1'\frac{1}{1+r}(\alpha_2'-\kappa_2')+\Gamma_2'(\frac{1}{1+r}(\alpha_1'+\kappa_1')-F+\gamma_X-\gamma_{\bar{X}})\right). \tag{94}$$

Subtract $F(\frac{1}{\frac{1}{1+r}}+L')$ from both sides and add $F(L+SL_S)$ to both sides to obtain

$$\pi - rF - FL(S',D') = F(L(S,D)+SL_S(S,D)-L(S',D'))+m(\alpha_2-\kappa_2)-\kappa_1+\frac{1}{\frac{1}{1+r}}({}'\gamma_{\bar{X}} -{}' \gamma_X)+\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}$$
$$\left(\Gamma_1'\frac{1}{1+r}(\alpha_2'-\kappa_2')+\Gamma_2'(\frac{1}{1+r}(\alpha_1'+\kappa_1')-F+\gamma_X-\gamma_{\bar{X}})\right) \tag{95}$$

$$\implies \xi(S',D') = \underbrace{L_S(S,D)SF+\big(L(S,D)-L(S',D')\big)F}_{\text{Congestion channel}}+\underbrace{\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}\Gamma_2'(\frac{1}{1+r}\alpha_1'-F)}_{\text{Pollution persistence channel}}+\underbrace{\frac{1}{1+r}\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}\Gamma_1'\alpha_2'}_{\text{Pollution hazard channel}}$$
$$\underbrace{+m\alpha_2}_{\substack{\text{Pollution}\\\text{hazard}\\\text{channel}}}+\underbrace{\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}\left(\Gamma_2'(\frac{1}{1+r}\kappa_1'+\gamma_X-\gamma_{\bar{X}})-\frac{1}{1+r}\Gamma_1'\kappa_2'\right)-(m\kappa_2+\kappa_1)+\frac{1}{\frac{1}{1+r}}(\gamma_{'\bar{X}}-\gamma_{'X})}_{\text{Adjustments for prior or upcoming corner solutions}}. \tag{96}$$

Along an interior launch path, the MEC $\xi(S',D')$ reduces to

$$\xi(S',D') = L_S(S,D)SF+\big(L(S,D)-L(S',D')\big)F+\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}\Gamma_2'(\frac{1}{1+r}\alpha_1'-F)+\frac{1}{1+r}\frac{\Gamma_1+m\Gamma_2}{\Gamma_1'+m\Gamma_2'}\Gamma_1'\alpha_2'+m\alpha_2, \tag{97}$$

and in an interior steady state the MEC further reduces to

$$\xi(S,D) = L_S(S,D)SF+\Gamma_2(\frac{1}{1+r}\alpha_1-F)+\frac{1}{1+r}(\Gamma_1+m)\alpha_2. \tag{98}$$

B.3 The collision probability and new fragment formation functions

In this section we derive the functional forms of the collision probability and new fragment functions, discuss the physical assumptions they encode, and describe our process for calibrating the physical model in more detail.

For numerical simulations, we model the probability that objects of type j are struck by objects of type k as

$$p_{jk}(k_t) = 1 - e^{-\alpha_{jk} k_t}, \tag{99}$$

where $\alpha_{jk} > 0$ is a physical parameter ("intrinsic collision probability") reflecting the relative mean sizes, speeds, and inclinations of the object types (see Letizia (2016) for a derivation of the physical content of α_{jk}). The probability a satellite is destroyed is the sum of the probabilities it is struck by debris and by other satellites, adjusted for the probability it is struck by both. For satellite-satellite and satellite-debris collisions, equation 99 gives us

$$\begin{aligned} L(S, D) &= p_{SS}(S) + p_{SD}(D) - p_{SS}(S)p_{SD}(D) \qquad (100)\\ &= (1 - e^{-\alpha_{SS}S}) + (1 - e^{-\alpha_{SD}D}) - (1 - e^{-\alpha_{SS}S})(1 - e^{-\alpha_{SD}D}) \\ \implies L(S, D) &= 1 - e^{-\alpha_{SS}S - \alpha_{SD}D}. \qquad (101) \end{aligned}$$

We write the new fragment formation function as

$$G(S, D) = F_{SD}p_{SD}(D) + F_{SS}p_{SS}(S) + F_{DD}p_{DD}(D), \tag{102}$$

where F_{jk} is the number of fragments produced in a collision between objects of type j and k. Letting $F_{SS} = \beta_{SS}S$, $F_{SD} = \beta_{SD}S$, and $F_{DD} = \beta_{DD}D$ where $\beta_{jk} > 0$ is a physical parameter reflecting the physical compositions and masses of the colliding objects, and using the forms in equation 99, we obtain

$$G(S, D) = \beta_S S(1 - e^{-\alpha_{SS}S})S + \beta_{SD}(1 - e^{-\alpha_{SD}D})S + \beta_{DD}(1 - e^{-\alpha_{DD}D})D. \tag{103}$$

The form in equation 101 is convenient as it allows us to solve explicitly for the open access launch rate and is easy to manipulate. Similar forms have been used in engineering studies of the orbital debris environment, and are currently used by the European Space agency in developing indices to study the long-term evolution of the orbital environment (Letizia, 2016; Letizia et al., 2017; Letizia, Lemmens, and Krag, 2018).

To derive equation 101, we consider balls (satellites and debris) being placed into bins (the set of all possible orbital paths within the shell of interest). The probability of a specific satellite being struck by another object is then equivalent to the probability that a randomly-placed ball ends up in a bin containing the specific ball we are focusing on. This is a version of the "pigeonhole principle", used in Béal, Deschamps, and Moulin (2020) to derive a similar form for satellite-satellite collisions.

Suppose we have b equally-sized bins and $n+1$ balls in total, where $b \geq n+1$. Without loss of generality, we label the ball we are interested in as i. We will first place i into an arbitrary bin, and then drop the remaining N balls into the b bins with equal probability over bins. The probability a ball is dropped into a given bin is $\frac{1}{b}$, and the probability a ball is not dropped into a given bin is then $\frac{b-1}{b} = 1 - \frac{1}{b}$. As we drop the remaining n balls, the probability that none of the balls is dropped in the same bin containing j is

$$Pr(\text{no collision with } i) = \left(1 - \frac{1}{b}\right)^n \tag{104}$$

Consequently, the probability that any of the n balls are dropped into i's bin is

$$Pr(\text{collision with } i) = 1 - \left(1 - \frac{1}{b}\right)^n. \tag{105}$$

Now suppose we are interested in the probability that members of a collection of j balls, $1 \leq j < b$, end up in a bin with one of the remaining $n+1-j$ balls. The probability that any of the remaining balls end up in a bin with any of the j balls we are interested in is then

$$Pr(\text{collision with } i) = 1 - \left(1 - \frac{j}{b}\right)^{n+1-j}. \tag{106}$$

As the number of bins and balls grow large ($\lim_{b,n\to\infty}$), we obtain

$$Pr(\text{collision with } i) = 1 - e^{-j}. \tag{107}$$

Though neither the number of objects in orbits nor the possible positions they could occupy is infinite, the negative natural exponential form is likely a reasonable approximation. If we suppose that we have two types of balls j and k of different sizes and bins the size of the smallest type of ball, we get that the probability a ball of type k is dropped into in a bin with a ball of type j as

$$Pr(k\text{–}j \text{ collision}) = 1 - \left(1 - \frac{\alpha_{jk}k}{b}\right)^{n+1-k} \tag{108}$$

$$\implies \lim_{b,n\to\infty} Pr(k\text{–}j \text{ collision}) = 1 - e^{-\alpha_{jk}k}, \tag{109}$$

which is the form in equation 99, where α_{jk} is a nonnegative parameter indexing the relative sizes of objects j and k. In the orbital context, α_{jk} reflects not only the sizes of the objects but also their relative speeds and inclinations. From here we obtain the form of L by applying standard rules of probability to satellite-satellite and satellite-debris collisions. Equation 103 follows from the form of L.

This "kinetic gas-like" approximation is used extensively in the space debris modeling literature as a tractable approximation of results from more complex and computationally-intensive orbital mechanics simulators. It is most suitable for long-term modeling studies with "large" (relative to the timescale of orbital interactions) time steps. As described in Letizia (2016), this approximation is equivalent to modeling collisions as a Poisson process. The Poisson assumption that the number of events occurring in non-overlapping time intervals are independent is equivalent to assuming that objects move randomly throughout the shell volume. This assumption is clearly not true, leading to our regularization approach described below. The assumption that the probability of an event is proportional to the length of the interval implies that fragment clouds are dispersed enough, and contain enough fragments, to be considered a continuum. Since our model is solved at annual timesteps while debris clouds evolve at much smaller timescales, this assumption is reasonable for our purposes.

B.4 Modeling debris growth over the next century

As we note in the main text, truly "unbounded" growth is unphysical, as collisional activity will reduce the fragments to smaller sizes and objects in LEO will eventually decay due to drag, solar radiation pressure, and other orbital perturbations. However, we follow the existing engineering literature on source-sink evolutionary models of the debris environment in allowing unbounded growth over the next century (Talent, 1992; Lewis et al., 2009; Lifson et al., 2022). An example using empirical data from a collision and the size-energy scaling law may help illustrate the underlying reasoning for this modeling choice.

Consider a collision between two large intact bodies, e.g. an event like the Iridium-Cosmos collision on February 10, 2009. Iridium 33 was an operational US communications satellite (S) while Cosmos 2251 was defunct Russian communications satellite (D). The table below from Kelso et al. (2009) shows the relevant size and mass characteristics of the initial objects and resulting fragments.

Table 1: "Table 1. Pre-Collision Satellite Characteristics." from Kelso et al. (2009)

Satellite	Number of Pieces	Total Volume (m^3)	Dry Mass (kg)	Inclination (deg)
Iridium 33	386	3.388	556	86
Cosmos 2251	927	7.841	900	74

The average radii for fragments from Iridium and Cosmos were around 12.8 cm and 12.6 cm, with average masses around 1.44 kg and 0.971 kg. These figures imply that the tracked fragments larger than 10 cm radius account for most of the initial body masses.[31]

The relation between a uniform sphere's kinetic energy and mass, given density ρ and velocity v, is

$$KE(r) = \frac{1}{2} \underbrace{\rho \left(\frac{4}{3} \pi r^3 \right)}_{\text{mass = density} \times \text{volume}} v^2. \tag{110}$$

Suppose a fragment of around 10 cm radius is a uniform aluminum sphere—a common assumption in debris modeling given the prevalence of aluminum in satellite construction, e.g. Letizia (2016). Aluminum has a mass of around 2.7 g/cm^3, giving a volume of 4188 cm^3 and mass of around 11 kg. Typical objects in low-Earth orbit have velocities on the order of 10 km/s (Lifson et al., 2022; D'Ambrosio et al., 2023).[32] Such a fragment will therefore have a kinetic energy of roughly 550 megajoules , or approximately 131 kg of TNT (energy equivalent of 1 kg of TNT is 4.184 megajoules). This is in the category of "hypervelocity" impacts that can shatter the intact object

[31] 10 cm is also the lower detection limit for sensor systems, raising concerns about censoring. The mass accounting suggests censoring may not be quantitatively large.

[32] Velocity in orbit is linked with altitude—accelerating or decelerating along its forward direction raises and lowers altitude, respectively.

(?). If the object is like Iridium or Cosmos—not-atypical LEO satellites—it may produce hundreds of fragments.

Since mass scales cubically with object radius, a reduction in average fragment size to 1 cm radius reduces the mass to 0.011 kg, producing an impact energy of 0.55 megajoules—comparable to the force of a hand grenade (European Space Agency, 2023). Even if it takes tens of collisions with fragments of 1-10 cm radius to overcome shielding on a large intact object, the resulting tens or hundreds of fragments will ensure net growth. To the extent that these objects move in debris "fields"—which may occur systematically due to orbital mechanics factors, particularly when a larger body is struck by a smaller one, e.g. Oltrogge et al. (2022); Oltrogge, Alfano, and Hall (2022); Pardini and Anselmo (2023)— their lethal effects at these and even smaller sizes may be amplified.

Suppose we take 1 cm to be a conservative "lethal size limit". How long will it take for collisional activity to reduce a fragment below this limit? Suppose the average cumulative annual collision probability for an arbitrary debris fragment is 25%—perhaps a high estimate, but again erring on the side of caution. That fragment will go roughly 4 years between collisions. If fragments are reduced to roughly 1 cm radius after only two collisions, it would take about 8 years for that debris fragment and its children to be rendered nonlethal. At 1% collision probability, the fragment's lethal lifetime is around 200 years.

At 575 km altitude, a large intact object has a residence time (i.e. time before it falls back to Earth due to drag) on the order of 10 years, and a 10 cm fragment has a residence time on the order of a year, for an upper bound on lethal lifetime of around 11 years. At 775 km altitude, the residence times are around 190 years for an intact object and 10 years for a fragment, for an upper bound on lethal lifetime of around 200 years. During their residence times the objects slowly drift downwards, entering lower shells. Most satellites are currently near or above 575 km altitude. Since plausible lethal lifetimes are on the order of relevant residence times, debris are likely to spend most of their lives at lethal sizes. Given a sufficiently large amount of mass at currently-popular altitudes (e.g. 100,000 satellites at 250 kg each spread over 550-800 km altitude), it seems reasonable to consider potential growth to "unbounded" levels over the next century.

B.5 Open access with a finite horizon

We employ an infinite-horizon modeling approach in the general model. However, one may reasonably wonder whether our conclusions regarding the open-access equilibrium are sensitive to this point. In this section we show that a finite-horizon problem with terminal period T (where it either becomes prohibitively costly to use the volume or Kessler Syndrome occurs or both) produces the same equilibrium condition.

Suppose there exists a final period, T, such that the potential launchers will all exit the market. We are agnostic as to why this may be the case, except to note that if such a period exists, it must be that there are no profits to be gained from launching after that period. In the final period, the launcher's value becomes

$$V_{iT}(S_T, D_T, X_T) = \max_{x_{iT}\in\{0,1\}} \{(1-x_{iT})\frac{1}{1+r}V_{iT+1}(S_{T+1}, D_{T+1}, 0)+x_{iT}\left[\frac{1}{1+r}Q(S_{T+1}, D_{T+1}) - F\right]\}. \tag{111}$$

There are two possible cases here for the value of launching in the final period, $\frac{1}{1+r}Q(S_{T+1}, D_{T+1})-F$:

1. $\frac{1}{1+r}Q(S_{T+1}, D_{T+1}) - F = 0$. In this case the potential launchers are indifferent between launching in the final period or not launching. By backwards induction the equilibrium path up to period T will match the one derived in the general model in the main text, with equation (15) being the equilibrium condition.

2. $\frac{1}{1+r}Q(S_{T+1}, D_{T+1}) - F < 0$. In this case, firms would prefer not to launch. Optimization by individual launchers therefore implies $V_{iT}(S_T, D_T, X_T) = 0$. This matches equation (14), which yields (15) after some algebra. So again by backwards induction the equilibrium path up to period T will match the one derived in the general model in the main text.

Indeed, it is possible to go one step further: the existence of such a terminal period (where $X_t = 0\ \forall t \geq T$) is possible if and only if Kessler Syndrome occurs along the equilibrium path.

Proposition 4. *A terminal period T where $\hat{X}_t = 0\ \forall t \geq T$ can exist for an open-access equilibrium path $\{\hat{X}_t\}_t$ if and only if Kessler Syndrome occurs (i.e. $\lim_{t\to\infty} D_t = \infty$) along the open-access equilibrium path.*

Proof. The proposition asserts that

$$X_t = 0 \ \forall t \geq T \iff \lim_{t\to\infty} D_t = \infty \tag{112}$$

We first show the $\Longleftarrow$ direction, then the $\Longrightarrow$ direction.

The "only if" direction, $X_t = 0 \ \forall t \geq T \Longleftarrow \lim_{t\to\infty} D_t = \infty$**:** If $\lim_{t\to\infty} D_t = \infty$, then there is some period $\bar{t}$ such that $D_t > D_{\bar{t}}$ for all $t > \bar{t}$. From the law of motion for D and our assumption that $\lim_{t\to\infty} D_t = \infty$, we can see that D_t must be monotonically increasing after $\bar{t}$. So there must exist a period $T \geq \bar{t}$ such that $L(S_t, D_t)F > \pi - rF$ for all $t \geq T$, i.e. where it becomes unprofitable to launch one more satellite at that or any future period. Thus, $X_t = 0 \ \forall t \geq T$. This completes the $\Longleftarrow$ direction.

The "if" direction, $X_t = 0 \ \forall t \geq T \Longrightarrow \lim_{t\to\infty} D_t = \infty$**:** If $X_t = 0$ for all $t \geq T$ then it must be the case that $\frac{1}{1+r}Q(S_{t+1}, D_{t+1}) - F < 0$ for all $t \geq T$, else some firm would find it profitable to launch. Note that it must be unprofitable to launch at t given that there are *no* launches occurring at t.

To be explicit in the next steps, we write the satellite and debris stocks with the previous-period aggregate launch rate X_t shown explicitly as an argument, i.e. writing $S_{t+1}(X_t)$ and $D_{t+1}(X_t)$. Along a path $\{S_t(0)\}_{t>T}^{\infty}$, clearly $S_{t+1}(0) \leq S_t(0)$. Now, $\frac{1}{1+r}Q(S_{t+1}(0), D_{t+1}(0)) - F < 0$ for all $t \geq T$ implies that $L(S_{t+1}(0), D_{t+1}(0))F > \pi - rF$ for all $t \geq T$. Monotonicity of L and $S_{t+1}(0) \leq S_t(0)$ then imply that $D_{t+1}(0) \geq D_t(0)$.

If there exists a threshold D^K such that $\lim_{t\to\infty} D_t(X_t) = \infty$ when $D > D^K$ for any X_t, then there are only two possible cases: either $\lim_{t\to\infty} D_t(0) < D^K$, or $\lim_{t\to\infty} D_t(0) \geq D^K$. The first case is a contradiction when G is strictly convex increasing, as each increase in $D_{t+1} - D_t$ must be larger than $D_t - D_{t-1}$ so eventually D_t must exceed D^K. Only the second case is consistent with the general physical model. This completes the $\Longrightarrow$ direction. □

Finally, how large is the volume available to be filled? Recent analyses estimate the maximum capacity consistent with stable orbital populations (i.e. no Kessler Syndrome) between 200-900km altitude to be on the order of 1.8 million active satellites, assuming no debris (Lifson et al., 2022). Over the next few decades, the total number of objects slated for launch is expected to be on the order of 80,000 satellites (Patel, Samira and Koller, Josef S., 2022). It is unclear whether there is sufficient demand to support hundreds of thousands of satellites, let alone over a million. While we do not think the maximum capacity described in the engineering literature will be realized due to both the externalities described here and in the economic literature and the aforementioned demand limitations, the large capacity available makes the issue seem less one of filling the volume with satellites or debris than one of operating in the volume becoming too costly due to risk.

C Calibration details

C.1 Data

We calibrate the economic parameters of our model using data collected by The Space Report (Space Foundation, 2021) on the annual revenues accruing to each sector of the space economy from 2006-2019. These data have been used in other economic analyses of space and orbit use (Wienzierl, 2018; Rao, Burgess, and Kaffine, 2020; Crane et al., 2020; Rao and Letizia, 2021). The data are not ideal for our purpose as they are aggregates covering the entire space sector, but more granular datasets describing specific LEO satellite operators' revenues and costs are not available. To focus on revenues and costs relevant to LEO satellite operators, we use only the variables which are plausibly attributable to LEO satellite activities. We calculate total LEO satellite operator revenues as the sum of the "Satellite communications" and "Earth observation" variables, and total LEO satellite operator costs as the sum of the "Ground stations and equipment", "Space Situational Awareness" (SSA), "Insurance premiums", "Commercial satellite launch", and "Commercial satellite manufacturing" variables. We discard variables representing revenues to the direct-to-home television, GNT (Geolocation, Navigation, and Timing), and satellite radio sectors, as these are provided by satellites in higher orbits beyond LEO. We also exclude suborbital commercial human spaceflight deposits as they are by definition for transit to regions below orbital altitudes (e.g. 50-80 km above mean sea level). Since our data is recorded annually, we set the period length to 1 year. We display the calculated variables in table 2. Note that these are *not* the revenues and costs accruing specifically to LEO operators—a distinction not possible given our data. Rather, these variables represent a superset of LEO operator revenues and costs, as they necessarily include some geostationary satellites. We describe our strategy to account for this issue during calibration in Appendix C.2.

Table 2: Economic data. Figures are in nominal billion USD. Data from Space Foundation (2021) and authors' calculations.

Year	Maximum total revenues attributable to all operators potentially using LEO	Maximum total costs attributable to all operators potentially using LEO
2006	13.800	80.840
2007	16.368	92.956
2008	18.104	85.371
2009	18.695	69.270
2010	19.570	68.460
2011	21.424	83.853
2012	22.747	93.779
2013	23.683	108.199
2014	24.002	127.567
2015	25.884	87.222
2016	26.087	89.201
2017	26.545	95.857
2018	28.420	99.930
2019	27.320	119.160

We calibrate physical parameters of our model using a kinetic gas approximation of orbital mechanics and data from DISCOS (Letizia et al., 2017; European Space Agency, 2021). These data describe the launch traffic, active satellites, and tracked debris objects (i.e larger than 10 cm diameter) in the 600-650 km shell over the 2006-2020 period. These data aggregate over different types of operators (e.g. commercial operators, civil government operators, defense operators). We display these data in table 3, along with the collision probability calculated from the kinetic gas approximation assuming satellite operators avoid 99% of all collisions between satellites and 95% of all collisions between satellites and tracked debris. Letting the avoidance success rates be κ_{SS}

and κ_{SD}, the probability of an unavoidable collision becomes

$$L(S,D) = (1-\kappa_{SS})(1-e^{-\alpha_{SS}S})+(1-\kappa_{SD})(1-e^{-\alpha_{SD}D})-(1-\kappa_{SS})(1-\kappa_{SD})(1-e^{-\alpha_{SS}S})(1-e^{-\alpha_{SD}D}). \tag{113}$$

Many ostensibly-non-commercial satellites are operated as joint ventures with commercial enterprises and many commercial satellite operators serve primarily civil government or defense customers, so we do not separate the satellite data by operator type. Further, since all satellites contribute to debris and collision probability regardless of their operator type, non-commercial operators' satellites ought to be included in the state vector. Non-commercial operators may also contribute to the observed "occupancy elasticity" (described precisely in the following section), further complicating efforts to properly disentangle payoffs to different operator types from the available data.

The DISCOS physical data also provide object characteristics such as mass and cross-sectional area, which are necessary for the kinetic gas approximation. We describe the details of the kinetic gas approximation of orbital mechanics in Appendix C.3.

Table 3: Orbital traffic in the 600-650 km shell. Collision probability is rounded. Data from European Space Agency (2021) and authors' calculations.

Year	Satellites launched	Active satellites satellites	Tracked debris	Collision probability
2006	15	25	211	2.95×10^{-6}
2007	84	31	275	3.84×10^{-6}
2008	168	47	273	3.85×10^{-6}
2009	72	43	393	5.48×10^{-6}
2010	156	53	444	6.20×10^{-6}
2011	30	56	411	5.76×10^{-6}
2012	73	53	429	6.00×10^{-6}
2013	213	64	454	6.37×10^{-6}
2014	261	97	484	6.87×10^{-6}
2015	175	122	495	7.09×10^{-6}
2016	15	114	494	7.05×10^{-6}
2017	26	122	525	7.49×10^{-6}
2018	36	139	506	7.28×10^{-6}
2019	33	155	543	7.83×10^{-6}
2020	9	158	626	8.97×10^{-6}

C.2 Economic calibration

To calibrate our economic model, we make three modifications to the open-access equilibrium condition in equation (17). First, we allow the per-period satellite payoff and cost to vary over time, i.e $\pi \to \pi_t$ and $F \to F_t$. This changes the equilibrium condition to

$$\pi_{t+1} = (1+r)F_t - (1 - L(S_{t+1}, D_{t+1}))F_{t+1} \tag{114}$$

$$\implies L(S_{t+1}, D_{t+1}) = 1 + \frac{\pi_{t+1}}{F_{t+1}} - (1+r)\frac{F_t}{F_{t+1}}. \tag{115}$$

This form is similar to the one described in equation (17) but for the time subscripts and term $1 - (1+r)\frac{F_t}{F_{t+1}}$. This term represents capital gains accruing to a period t launcher from increases in the cost of building and launching a satellite in period $t+1$. We abstract from operators' expectations over economic variables and assume they perfectly forecast all $t+1$ objects.

Second, we allow the per-period satellite payoff to depend on the current stock of satellites in orbit, i.e $\pi_t \to p_t(S_t)$. We use a constant elasticity form with exponential factor productivity growth, $p_t(S_t) = \pi e^{at}(1+\eta)S_t^{\eta}$, where η is the "orbital occupancy elasticity of per-period satellite payoffs". We assume that the downstream market for satellite outputs is competitive such that operators do not internalize $\frac{\partial p_t}{\partial S_t}$. The equilibrium condition becomes

$$L(S_{t+1}, D_{t+1}) = 1 + \frac{p_{t+1}(S_{t+1})}{F_{t+1}} - (1+r)\frac{F_t}{F_{t+1}}. \tag{116}$$

Third, we incorporate exogenous limited satellite lifespans to allow for natural depreciation and replacement of satellites. Specifically, we assume each satellite is replaced with probability μ each period. We calibrate this value explicitly to simulate object stocks (described in the Appendix C.3); for now, we leave this to be adjusted in the regression-based calibration approach described below. The final equilibrium condition for our simulations is

$$L(S_{t+1}, D_{t+1}) = 1 + \frac{1}{1-\mu}\frac{p_{t+1}(S_{t+1})}{F_{t+1}} - \frac{1+r}{1-\mu}\frac{F_t}{F_{t+1}}. \tag{117}$$

To simulate future periods under different returns growth rate and occupancy elasticity assumptions, we estimate the growth rate of total LEO satellite operator costs. We estimate

$$\log(F_t) = \eta_0^F + \eta_1^F t + \nu_t^F, \tag{118}$$

where $\log(F_t)$ is the natural log of total LEO satellite operator costs, t is the year, the growth rate (the object of interest) is $g = \exp(\eta_1^F) - 1$, and the regression error is ν_t^F. The estimated growth rate is roughly 2.5%, which is consistent with Crane et al. (2020).

There are two final steps to our procedure: ensuring consistency between the occupancy elasticity and factor productivity parameters, and accounting for unobserved variables. To ensure consistency between the assumed elasticity and implied orbital slot factor productivity and match the final observed value of LEO-using sector revenues ($27.32b in 2019, see table 2), we calibrate the factor productivity term π in equation 25. Specifically, letting K be the observed value to match for each assumed elasticity value η_j, setting $t = 0$ and S_0 to the shell-specific initial condition ($S_0 = 158$), the factor productivity term π_j satisfies

$$\pi_j = \exp(\log(K) - \log(1 + \eta) + \eta \log(S_0)). \tag{119}$$

Finally, as mentioned in the previous section, using maximum total sector revenues and costs directly from the data in table 2 as though the data reflects only operators in the 600-650 km shell is challenging for two reasons. First, the data in table 2 cover all satellite operators—our variable selection step is the only thing restricting the set of operators included in the data. Even if we were successful in removing all operators outside of LEO through variable selection when calculating total LEO operator revenues and costs, the revenue and cost variables will still include operators outside the 600-650 km shell. The data aggregation implies an unobservable "shell-share" coefficient, $s \in [0, 1)$, scaling observed aggregate revenues and costs to reflect only the portion attributable to satellites in the 600-650 km shell. Second, theory predicts that the discount rate used by operators is a critical parameter in determining LEO use, but this parameter is unobserved.

Fortunately, equation 115 offers a way to address both challenges. Letting π_t be the total LEO satellite operator revenues and F_t be the total LEO satellite operator costs from table 2, and L_t be the collision probability shown in table 3, we estimate the following regression on data from

2006-2019:

$$L_t = \gamma_0 + \gamma_1 \frac{\pi_t}{F_t} + \gamma_2 \frac{F_{t-1}}{F_t} + e_t. \tag{120}$$

The estimated "adjustment coefficients" $(\gamma_0, \gamma_1, \gamma_2)$ reflect the shell-share s, the discount rate r, as well as the satellite turnover μ (though they are not separately identified). We use the adjustment coefficients to simulate the model in future periods given projected growth in π_t and F_t. If the shell-share coefficients (labeled s in the preceding discussion) are common to revenues and costs and time-invariant (or "close" and "slowly-varying"), they will (almost) cancel out of the ratios we use in equation (120) and our estimated adjustment coefficients would only reflect satellite turnover and discounting.[33]

C.3 Physical calibration

Here we describe key equations and the ridge regression approach to correcting for non-random object paths. Readers interested in detailed explanations of the physics-based elements of our calibration approach, including derivations and validation, are referred to Letizia (2016).

We require physically-appropriate values for the following parameters: δ, μ, α_{SS}, α_{SD}, α_{DD}, β_{SS}, β_{SD}, β_{DD}. Calibrating δ and μ (the mean debris decay rate and mean satellite active lifetime time) are the most straightforward. We take data from ESA regarding the residence time δ_r of debris objects and lifetime of active satellites μ_r at different altitudes (European Space Agency, 2021). We set the decay rate for debris objects as $\delta = \min\{1 - \delta_r^{-1}, 1\}$ and the natural turnover rate for satellites as as $\mu = \min\{1 - \mu_r^{-1}, 1\}$. For both parameters we calculate share-weighted averages across object types within the category to reflect the effects of heterogeneous object dimensions, e.g. δ reflects the weighted average of decay times for rocket bodies, fragments, and intact derelict objects.[34] We calculate the share-weighted decay rate in the 600-650 km shell is roughly 7% every year.[35] The share-weighted average active LEO satellite lifetime is roughly 6.71 years. This implies

[33] This is not the only interpretation of our estimates—as described in Rao, Burgess, and Kaffine (2020), the adjustment coefficients may also reflect unmodeled frictions in satellite launching and operation.

[34] Cross-sectional area and mass are key determinants of orbital residence times. Both can vary significantly within object classes.

[35] At these altitudes, the decay rates from higher shells rapidly approach zero. For the 650-700 km shell,

roughly 15% of active satellites in LEO turn over every year on average, i.e the fraction remaining is $1 - \mu = 0.85$.

We calibrate the parameters of L and G in two steps. First, we compute the collision probability and new fragment formation parameters using a kinetic gas approximation similar to the one used in Letizia et al. (2017) and Letizia, Lemmens, and Krag (2018) as well as analytical fragmentation formulas from Krisko (2011) and Letizia (2016) calibrated to the NASA standard breakup model. These formulas require data on object mass and cross-sectional area, which we obtain from DISCOS. The DISCOS parameters describe average values across different types of active satellites and debris objects, so we compute share-weighted averages for active satellites and debris objects. The kinetic gas approximation implies that objects within the shell are moving randomly, leading to our next step. Second, to adjust for the non-random motion of objects in the shell, we regularize the expected fragmentation components of G by estimating a ridge regression on the debris law of motion using data in table 3 and the analytically-computed parameter values. We also use this second step to jointly estimate the launch debris parameter m from the ridge regression. We describe our procedure for calibrating the physical model parameters in more detail in Appendix section C.3. Table 4 summarizes the calibrated parameter values.

To calculate the intrinsic collision probabilities $\alpha_{SS}, \alpha_{SD}, \alpha_{DD}$, we start with data regarding object cross-sectional areas for active satellites (commercial, military, civil government, and other) and intact debris objects. We assume debris fragments are uniform aluminium spheres of diameter 10 cm, and treat all other objects as uniform spheres as well. We compute the cross-sectional areas of active satellites and debris within each shell as share-weighted averages over 2006 2019 across the types of objects within each class, e.g. if 20% of the debris objects are intact and 80% are fragments we calculate the area as $0.2 * (intact\ area) + 0.8 * (fragment\ area)$. Under these assumptions the rate at which a reference object moving randomly at speed s in a closed space of volume V is struck by an object of cross-sectional area a is

$$\frac{sa}{V}, \tag{121}$$

the share-weighted average decay rate is roughly 5%, and for the 700-750 km shell the decay rate is 3%. We therefore neglect objects entering the 600-650 km shell from higher altitudes as they are unlikely to significantly change our results.

Table 4: Summary of calibrated parameter values for the 600-650 km shell. Values are rounded to the nearest integer or second non-zero decimal place.

Parameter	Value	Notes
η_1^F	0.025	Total costs growth parameter. Standard error is 0.009.
γ_0	3.35e-06	Equilibrium adjustment coefficient 1 (open-access capital gains).
γ_1	2.22e-05	Equilibrium adjustment coefficient 2 (gross satellite rate of return).
γ_2	-2.67e-06	Equilibrium adjustment coefficient 3 (open-access capital gains).
δ	0.074	Annual fraction of debris decaying to lower shell.
μ	0.15	Annual active satellite turnover rate
α_{SS}	2.73e-07	Satellite-satellite collision rate parameter.
α_{SD}	2.73e-07	Satellite-debris collision rate parameter.
α_{DD}	2.78e-07	Debris-debris collision rate parameter.
κ_{SS}	0.99	Fraction of satellite-satellite collisions successfully avoided.
κ_{SD}	0.95	Fraction of satellite-debris collisions successfully avoided.
$\tilde{\beta}_{SS}$	1,800	Expected number of fragments from satellite-satellite collision. (regularized).
$\tilde{\beta}_{SD}$	333	Expected number of fragments from satellite-debris collision. (regularized).
$\tilde{\beta}_{DD}$	327	Expected number of fragments from debris-debris collision (regularized).
m	0.013	Expected number of launch debris remaining in shell after 1 year (regularized).

where the volume is determined by the altitude and our assumption of that the space is a spherical shell, and the speed is determined by the altitude, the Earth's gravitational constant, and our assumption that the objects are uniform spheres.

To calculate the unadjusted fragmentation rates, we use data on average object masses from ESA along with a formula found to fit the high-fidelity NASA standard breakup model described in Krisko (2011). Letting the mass of the object struck be M, and assuming the object is shattered into uniform 10 cm spheres, the number of fragments from a catastrophic collision n is

$$n = 0.1M^{0.75}0.1^{-1.71}. \tag{122}$$

The only steps remaining are to adjust our estimate of the expected number of fragments from collisions for the non-random motion of objects in the shell, and to set the value of the launch debris parameter m. ODE-based engineering models of the debris environment use such adjustment coefficients based fitting the ODE model to results from many computationally-costly runs of high-fidelity orbital environment models, e.g. as in Somma et al. (2017); Somma (2019). This approach would be even costlier for our model, as the launch rate is endogenous, and would not provide a useful estimate of the launch debris parameter m. We instead perform the adjustment and estimate m jointly using historical data and ridge regression, a regularization technique used to improve out-of-sample predictive performance at the expense of in-sample fit. Ridge regression achieves this goal by exploiting the bias-variance trade-off, shrinking parameter values toward zero in exchange for reduced prediction variance (Hoerl, Kennard, and Hoerl, 1985; Zou and Hastie, 2005).

Since satellites are specifically coordinated to reduce collisions, the adjustment for non-random motion should involve shrinking the expected number of fragments from a collision (with the expectation taken over the probability of a collision) toward zero. Ridge regression achieves this goal. Additionally, ridge regression is often used when the number of variables is "large" relative to the number of observations or when parameter estimates are known to be noisy due to (for example) high degrees of collinearity. Our model and data satisfy the former condition (with 4 parameters to estimate from 14 observations), and our physical calibration approach (specifically the assumption that all objects are uniform spheres) causes collinearity in our collision probability values. Since our collision model prescribes the functional form of the collision probability as $(1 - \exp(-\alpha_{jk}k))$, the effect of non-random motion on new debris growth cannot be separately identified from α_{jk} and β_{jk}. This is convenient for our regression-based adjustment, since it allows us to pose the ridge regression as a linear model. Specifically, letting $\bar{x}$ denote a physically-calibrated parameter value, we estimate the following regression:

$$D_{t+1} - (1-\delta)D_t = \rho_{SS}\beta_{SS}(1 - \exp(-\alpha_{SS}S_t)) + \rho_{SD}\beta_{SD}(1 - \exp(-\alpha_{SD}S_t)) + \rho_{DD}\beta_{DD}(1 - \exp(-\alpha_{DD}D_t)) + m + \nu_t^D, \tag{123}$$

where $\rho_{SS}, \rho_{SD}, \rho_{DD}, m$ are parameters to be estimated and ν_t^D is the error term. The final regularized estimates of the fragmentation and launch debris parameters are shown in table 4 as $\tilde{\beta}_{SS}$, $\tilde{\beta}_{SD}$, $\tilde{\beta}_{DD}$, m.

D Algorithms for equilibrium and optimum

To describe how we generate initial guesses for the social planner's problem, it is useful to formally state a finite-horizon sequence version of the planner's problem. Letting T be the final period, the planner's finite-horizon sequence problem is

$$\max_{\{X_t, S_{t+1}, D_{t+1}\}_{t=0}^{T}} S_t Q(S_t, D_t, X_t) + \frac{1}{1+r} \sum_{\tau=t}^{T-1} \frac{1}{1+r}^{\tau-t-1} X_\tau \left(\frac{1}{1+r} Q(S_{\tau+1}, D_{\tau+1}, X_{\tau+1}) - F \right) \tag{124}$$

$$\begin{aligned}
\text{s.t. } Q(S_t, D_t, X_t) &= \pi + \frac{1}{1+r}(1 - L(S_t, D_t)) Q(S_{t+1}, D_{t+1}, X_{t+1}) \text{ if } t < T \\
Q(S_T, D_T, X_T) &= \pi \\
S_{t+1} &\leq S_t(1 - L(S_t, D_t)) + X_t \\
D_{t+1} &\geq D_t(1-\delta) + G(S_t, D_t) + mX_t \\
X_t &\in [0, \bar{X}] \quad \forall t \\
S_{t+1} &\geq 0, D_{t+1} \geq 0 \\
S_0 &= s_0, D_0 = d_0.
\end{aligned}$$

The guess generation in algorithm 1 uses a result from Easley and Spulber (1981), that optimal plans generated from solving a finite horizon problem with sufficiently-large T closely approximate infinite-horizon optimal plans. Algorithm 1 describes our solution procedure more precisely.

Generating the open-access policy function is much simpler. At each node on a grid over S and D values (e.g. $\mathscr{G}_1$ as in Algorithm 1), we solve the open-access condition

$$\pi - rF - L(S'(X,S,D), D'(X,S,D))F = 0 \tag{125}$$

for the open-access launch rate X.

To generate the phase diagrams, we use solved policy functions X to compute the evolution of the satellite and debris stocks at each grid node. More precisely, we compute $dS = (S'(X,S,D) - S)/h$ and $dD = (D'(X,S,D) - S)/h$ for a fixed positive value h. The value of h is chosen to make the plotting more stable; we use $h = 10$, but other values yield similar results. The nullclines are

Algorithm 1: Solve the planner's problem

Generate a sparse initial grid, $\mathscr{G}_0$, over $(S, D) \in \mathbb{R}_{[0,a]} \times \mathbb{R}_{[0,b]}, \quad a, b > 0$.

At each point on $\mathscr{G}_0$, solve program 124 with T equal to a large number. Larger is better; we use $T = 150$, which balances compute time with guess quality. This produces an initial guess on a sparse grid, $\tilde{v}_0$.

Using linear interpolation, "infill" $\tilde{v}_0$ (defined on $\mathscr{G}_0$) to v_0 (defined on $\mathscr{G}_1$). $\mathscr{G}_1$ has the same boundaries as $\mathscr{G}_0$ ($(S, D) \in \mathbb{R}_{[0,a]} \times \mathbb{R}_{[0,b]}$) but contains more points. This gives an initial guess defined on a denser grid.

Set δ to some large number (we use 10) and ϵ to some small number (we use 1% of the mean value of v_0). Set $i = 0$ and $W_0(S, D) = v_0$.

while $\delta > \epsilon$ **do**

  At each node in $\mathscr{G}_1$, solve program 22 with $W(S_{t+1}, D_{t+1}) = W_i(S_{t+1}, D_{t+1})$. Label the value function obtained as $W_{i+1}(S, D)$, defined over $\mathscr{G}_1$. We use linear interpolation to compute $W_i(S_{t+1}, D_{t+1})$ when (S_{t+1}, D_{t+1}) is between nodes of $\mathscr{G}_1$.

  $\delta \leftarrow ||W_i(S, D) - W_{i+1}(S, D)||_\infty$.

  i $\leftarrow$ i+1

end

plotted as the zero-isoclines of dS and dD.