Sustainable Investing in General Equilibrium*

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This version: February 15, 2023

Preliminary and incomplete. Comments welcome!

Abstract

We investigate under what conditions sustainable investing—financial investors who like (resp. dislike) holding assets that they believe have a positive (resp. negative) environmental/societal impact— alter the allocation of capital between clean and dirty firms in a dynamic general equilibrium framework. We show that investors with preferences for sustainable investing can have both a *composition* and a *scale* impact on output, but the magnitude, the direction, and the timing of the impacts crucially depend on the size of sustainable investors in terms of stock market wealth, their investment horizon, and the distribution of firms across sustainability scores. While sustainable investing is shown to eventually impact capital allocation in favor of clean firms in the long run, it might take a long time before the positive impact materializes.

Keywords: Sustainable Investing, Impact Investing, ESG, Portfolio Choice

JEL codes: E10, G11, O40, Q50

*We are thankful to the participants in the Macro Lunch Workshop at UCSD for their comments and suggestions.

1 Introduction

Sustainable investment is defined as investing in companies for purposes other than maximizing pecuniary returns, typically taking the form of environmental and/or social priorities. Over the last few years, the asset management industry has embraced the idea of reflecting environmental, social, and governance considerations (under the acronyms of ESG, CSR, SRI, etc.) in managing portfolios. Figure 1 reports the rise of sustainable funds in major financial markets worldwide, measured in numbers of funds that specialize on sustainable investing, and in value of assets under their management. The current scale of roughly $3 trillion and the accelerating positive trend over the last three years indicate that sustainable investing is on course to achieve a size with the potential for macroeconomic impact.

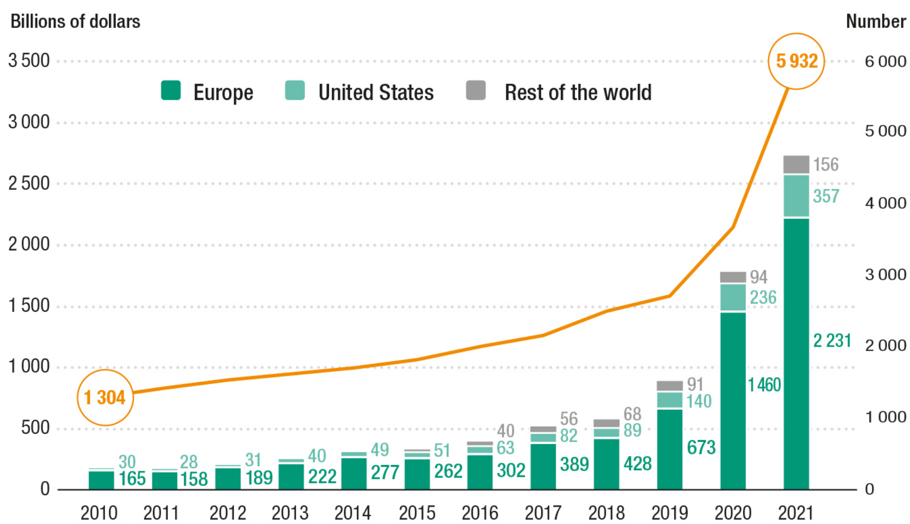

Figure 1: Sustainable Funds and Their Assets Under Management (AUM)

Figure 1 reports the numbers of investment funds that specialize on sustainable investing worldwide (yellow line, right scale), and the total value of assets under their management (columns, left scale). Sources: UNCTAD World Investment Report 2022, based on Morningstar data.

As a result of the growing interest of the financial industry in sustainable investing, an increasing number of academic studies have been documenting the impact of sustainable investing on asset returns and corporate behavior[1]. However, little attention has been devoted so far to the question of

[1] A non-exhaustive list of recent ones include Berk and van Binsbergen (2021); Gillan, Koch, and Starks (2021); Humphrey, Kogan, Sagi, and Starks (2021); Idzorek, Kaplan, and Ibbotson (2021); Kedia, Starks, and Wang (2021); Atz, Van Holt, Liu, and Bruno (2022); Berg, Koelbel, Pavlova, and Rigobon (2022); Caramichael and Rapp (2022).

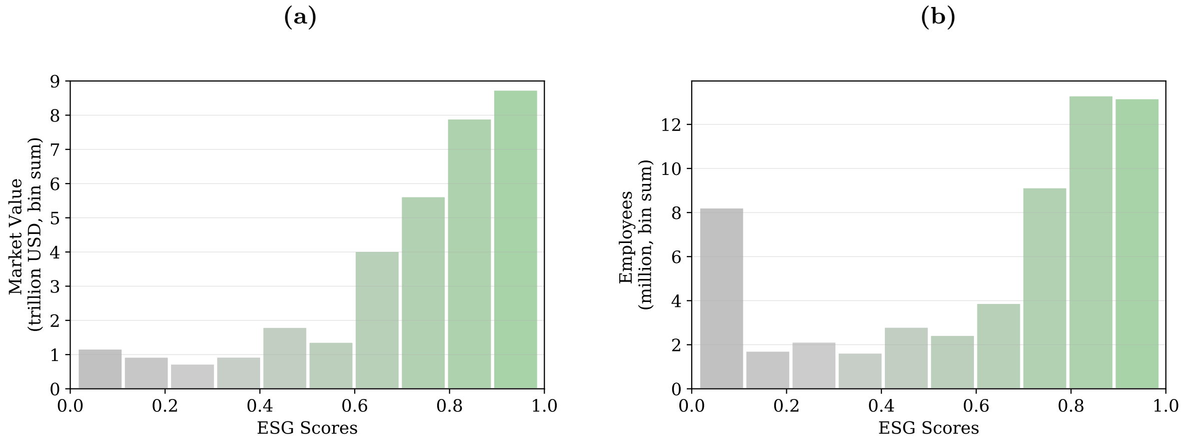

Figure 2: U.S. Publicly Traded Companies by Sustainability Score Deciles

Figure 2 reports the market capitalization (panel (a)), and the total employment (panel (b)) of U.S. publicly traded companies by Refinitiv sustainability score deciles. Companies that are not scored are assigned to the 0 bin.

how sustainable investing can impact the allocation of productive capital when general equilibrium adjustments are considered.[2]

In this paper, we study under what conditions sustainable investing impacts capital allocation across firms with different sustainability characteristics. We do so in the context of a dynamic general equilibrium model with heterogeneous investors and heterogeneous firms.

Investors are heterogeneous in terms of their preferences for sustainable investing: those who care about environmental or social values associated with their portfolio holdings are *advocate* investors, and those who are indifferent are *benchmark* investors. Firms, on the other hand, are heterogeneous in terms of the sustainability of their operations. Following the tradition of the environmental literature, we refer to firms with higher sustainability scores as *clean*, and those with lower score as *dirty*.

Figure 2 reports the distribution of U.S. publicly traded companies over the Refinitiv sustainability score, which ranges from 0 to 1. Panel (a) shows the market capitalization of firms, while panel (b) their total employment, both by score deciles. The message from both panels is that the majority of U.S. publicly traded firms sit at the higher end of the sustainability score distribution,

[2] A notable exception is Pástor, Stambaugh, and Taylor (2021), who study the impact of non-pecuniary preferences for clean stocks on the capital allocation of clean firms in the context of exponential utility two-period asset pricing model, but stop short of considering the implied dynamic general equilibrium adjustments. Early relevant contributions also include Beltratti (2005), Dam and Heijdra (2011), and Dam (2011).

at least as measured by Refinitiv.[3] The central question of our paper can be cast in quantitative form using the magnitudes in Figures 1 and 2. Let us assume for the sake of the argument that the U.S. publicly traded companies represented the universe of traded companies. The market capitalization of companies at the highest sustainability score in Figure 2 is \$9 trillion, so assuming that all the funds of sustainable investors, roughly \$3 trillion, are used to purchase stocks in those companies, it would amount to holding roughly 1/3 of their market value. How would that translate into changes in the cost of capital and thus productive investment for those companies, and for the economy as a whole, if we take into account the general equilibrium effects of the resulting portfolio reallocation? This is the ultimate question we are after.

Our key assumption is that advocate investors prefer to hold stocks in clean firms and are thus willing to accept a lower pecuniary return on those stocks, while they dislike holding stocks in dirty firms and require a higher pecuniary return to hold those stocks.[4] Benchmark investors are indifferent between clean and dirty firms, and they exclusively care about pecuniary returns. In our model, the reallocation of an investor's portfolio towards stocks of clean firms impacts productive capital allocation differently depending on the return demanded by the marginal investor, whose identity is determined in equilibrium.

In particular, when the relative wealth of advocate investors is small, overall capital is higher compared to an economy with only benchmark investors, but it is equally distributed across firms. We say that impact investors have a positive *scale* effect on capital, but no *composition* effect. Intuitively, advocate investors are willing to hold stocks at a lower pecuniary return, but they do

[3] Eight major companies currently publish sustainability scores: ISS ESG, Moody's, MSCI, Refinitiv, RepRisk, Sustainalytics, S&P Global CSA, Truvalue Labs. The correlation between sustainability ratings across rating companies is imperfect. For instance, Berg, Koelbel, and Rigobon (2022) estimate the correlation to be between 0.38 and 0.71.

[4] Hong and Shore (2022) survey the recent evidence on non-pecuniary returns. Geczy, Stambaugh, and Levin (2005) find that mutual funds investing in socially-responsible companies tend to slightly underperform standard mutual funds. In incentivized experiments, Riedl and Smeets (2017) find that "social preferences" and "social signalling" explain socially responsible investment (SRI) decisions more than financial motives. By examining a salience shock in the U.S. mutual fund market, Hartzmark and Sussman (2019) present causal evidence that investors marketwide value sustainability beyond what would be explained by their expectation of future performance or risk, consistent with non-pecuniary motives influencing investors' decisions. Barber, Morse, and Yasuda (2021) show that investors derive non-pecuniary utility from investing in impact-oriented venture capital funds and are willing to sacrifice pecuniary returns. Bauer, Ruof, and Smeets (2021) observe two-thirds of pension fund holders in their field survey are willing to expand the fund's engagement with companies based on UN's Sustainable Development Goals, even when they expect engagement to hurt financial performance. Baker, Bergstresser, Serafeim, and Wurgler (2022) find that green municipal bonds are issued at premium. Pástor, Stambaugh, and Taylor (2022) find that the stocks of clean firms had negative alphas while the stocks of brown firms had positive alphas. Zerbib (2022) studies an asset pricing model with tastes for clean stocks and dislike for sin stocks and finds that the taste effect ranges from -1.12% to +0.14% for the different industries and the average exclusion effect is 2.79%, in terms of annual returns.

not have enough wealth to be the marginal investor. Because somebody has to hold those stocks – we are in general equilibrium after all – benchmark investors are the marginal investors, and they are willing to hold clean stocks only if they offer the same return as dirty stocks. Hence the lack of impact on composition. Advocate investors, however, are willing to save more in order to increase their holdings of clean firms. This creates an increase in the supply of savings, which lowers the cost of capital equally across firms, and thus increase capital in all firms, hence the positive scale effect.

As time progresses, advocate investors accumulate enough savings to eventually become the marginal investor in clean firms. At that point, the cost of capital of the clean firms decouples from that of the dirty firms, and capital accrues disproportionately to clean firms, while it drops for dirty firms. Impact investors now have both a scale and a composition effect on capital allocation. We show that in the steady state, advocate investors always hold all the stocks in the clean firms, and capital allocation is permanently tilted towards the clean firms. Hence, our model predicts that sustainable investing eventually impacts capital allocation in favor of clean firms. However, depending on the initial level of wealth of advocate investors, the reallocation might take a long time to realize.

Our results have important policy implications. On the one hand, market forces work to reallocate capital towards firms that are favored by advocate investors. On the other hand, for those forces to have a significant impact, advocate investors have to be large enough, and that might substantially delay their ultimate impact. To the extent that capital reallocation towards clean firms is socially desirable, our results suggest that policies that subsidize the holding of clean stocks for both advocate and benchmark investors might be socially desirable.[5]

Our model abstracts from many interesting dimensions that are likely to inject significant nuisance on the results just described, especially in terms of the policy implications highlighted above, such as endogenous technology choice, risk considerations in asset returns, preferences of sustainable consumption, etc. However, we believe that the core mechanism we isolate int the model - the notion that sustainable investing modifies the saving profiles of investors and leads to a time-varying impact on the scale and composition of capital allocation - will continue to play a

[5]The heterogeneity in investors' preferences can be considered a reflection of how investors internalized differently an externality that is created by the dirty firms, and impacts everyone's well-being. In that sense, the government intervention is justified on account of advocate investors' slow impact on the market returns to correct the externality.

central role also in the presence of additional realistic features.

2 Model Preliminaries

The central assumption in our modeling of sustainable investing is that households derive direct utility from holding stocks in firms that have different sustainability profiles. Specifically, we assume that a sustainability index of wealth, denoted by $\mathcal{G}$, enters the household's utility function.[6] We first describe our modeling of preferences and then provide a discussion of our assumption.

2.1 Sustainable Wealth in the Utility Function

Our baseline model is populated by a continuum of firms indexed by a sustainability score $g \in [0, 1]$, with density $\phi(g)$. Let $q(g)$ denote the price of a share in firm g at a given point in time, and let $z(g)$ denote the stock holdings in firm g by the household. We postulate that the sustainable wealth index $\mathcal{G}$ takes the linear form[7]

$$\mathcal{G} = \int_0^1 v(g)z(g)q(g)\phi(g)dg. \tag{1}$$

The function $v(.)$ is assumed to be weakly increasing and bounded for $g \in [0, 1]$. The value $v(g)$ represents the impact on the sustainability wealth index of a change in the value of the holdings of firms with score g, modulo $\phi(g)$. We assume that the function $v(.)$ implies the existence of a "neutral" sustainability score g_n such that

$$v(g_n) = 0 \quad \text{for some} \quad g_n \in [0, 1]. \tag{2}$$

For firms with $g > g_n$, sustainable wealth is weakly increasing in their stock holdings, while for firms with $g < g_n$, sustainable wealth is weakly decreasing. Following the terminology frequently used in the environmental literature, we will refer to the former as *clean* firms/stocks, and to the latter as *dirty* firms/stocks.

We assume that utility is separable in consumption and sustainable wealth, and throughout

[6] Modeling wealth in the utility function has a long tradition in macroeconomics, see Kurz (1968). Recent examples include Michaillat and Saez (2021), and Mian, Straub, and Sufi (2021).

[7] The choice of a functional form for the index is made for analytical convenience and it carries no substantial implications for the qualitative nature of our results. One key advantage of a linear index is allowing for zero stock holdings, i.e. $z(g) = 0$ for some g, while maintaining the optimization problem well-defined. Furthermore, because the holdings of stocks are bounded, the index is well-defined even when $z(g)$ is at the upper bound for all g's.

the analysis, we assume log-utility for consumption, so flow utility is

$$\ln c + \mathcal{V}(\mathcal{G}), \tag{3}$$

where $\mathcal{V}$ is differentiable and weakly increasing in sustainable wealth, $\mathcal{G}$.

2.2 Discussion of Preferences The introduction of preferences for sustainable wealth can be thought to capture the internalization of some type of externality that is generated by firms in their operations. The simplest example is an economy in which firms produce an identical output good, but they do so by emitting different amounts of carbon dioxide to the atmosphere per unit produced. Carbon emissions, in turn, reduce the quality of the environment inhabited by households, thus reducing welfare.[8] A social planner in that economy would allocate capital, and thus output production, across firms taking into account the negative externality due to carbon emission. In the decentralized economy, if households have access to the information on carbon emissions for firms, and understand the negative impact, they might choose to internalize the externality by changing their holdings of firms according to their environmental impact, indicated by g.

A complementary way to model the preferences for sustainability is to assume that households can differentiate the consumption depending on which firm has produced it. By directing their expenditure towards sustainable firms, households increase the marginal product of capital, and thus can change the allocation in equilibrium. We do not consider that channel in our model because we want to isolate the impact of the allocation of savings across firms, and thus obtain predictions on how wealth and portfolio allocation translate into impact of sustainable investing.

2.3 Households/Investors We assume that the economy is populated by two types of households/investors: *advocate*, denoted as type "A," and *benchmark*, denoted as type "B." Households of type A care about the sustainability of their wealth portfolio, and thus have the type of preferences in (1)-(3) with $v(g) \neq 0$ for some $g \in [0, 1]$. Households of type B only care about the

[8] The channel that we have in mind here is what is typically considered by the literature on the macroeconomics of sustainability, for instance, see Heal (1998), where the stock of environmental assets is modeled as directly entering the utility function of the social planner. The goal is to capture in reduced form the many complex ways in which environmental assets contribute to welfare via their participation in the production process, and as amenities directly enjoyed by humans.

pecuniary return on their wealth, which corresponds to setting, $v(g) = 0$ for all $g \in [0, 1]$, in their preferences. We assume that there is one representative household per each type in the economy. This assumption is without loss of generality for our purposes because the relative size of each household will be modeled via the initial wealth that they hold in terms of stocks.

2.4 Firms We assume that firms all produce output from capital using the same technology, $f(k)$, which is strictly concave, increasing and differentiable. Firms own the capital and decide how much of their earnings should be reinvested or distributed as dividends, with the objective of maximizing the total stock market value of the firm.[9] The initial capital held by each firm is $k_0 > 0$.

Firms cannot issue new shares or debt, so their only source of financing is retained earnings. In particular, firms cannot directly sell capital to other firms, but they can adjust capital holdings via higher or lower retained earnings. This assumption limits the ability of firms to exploit possible arbitrage opportunities when they have access to lower cost of capital, and would want to sell that capital to firms that have access to a higher cost of capital. In equilibrium, firms that operate with a different marginal product of capital can therefore coexist. We interpret this assumption as a reduced form modeling of persistent impact of access to heterogeneous financial conditions, such technological innovations that are not necessarily portable across firms.

Throughout the analysis, we maintain the technology used by firms to produce output from capital into output identical across all g's. The identity of firms matters only insofar as they are perceived differently by households, which in turn may impact their cost of capital $\theta(g)$. We consider this channel to be at the heart of understanding the impact of sustainable investing. We explore the interaction between differences in productivity, and different usage of renewable and non-renewable resources, and preferences for sustainable investing, in a companion paper.

Finally, we implicitly assume that the size of a firm only depends on capital k. In particular, we do not consider labor as a variable/movable production input. As such, a firm in our model can be thought as corresponding to a worker/entrepreneur that is endowed with technology f, and capital k. The density $\phi(g)$ of firms at a given sustainability score g can then be interpreted as the

[9] We follow Brock and Turnovsky (1981) and Turnovsky (1990) in our modeling of the problem for the firm. The assumption that firms maximize shareholders value is not without loss of generality when the firm can issue new stocks, or bonds, but it corresponds to profit maximization under our maintained assumptions.

share of employment in firms with score g over total employment.

3 A Two-Period Model

We begin by analyzing a two-period model having the features outlined above. The equilibrium characterization of the two-period model contains useful predictions for the analysis of the fully dynamic model.

3.1 Firms and Households

The objective function for firms at time 0 is specified as,

$$V \equiv q + d_0, \tag{4}$$

where q denotes the ex-dividend price of a share at time 0, and d_0 is the per-share dividend paid also at time 0. We assume that capital fully depreciates when used in production, so the maximization is subject to the constraints

$$d_0 \leq f(k_0) - k, \quad \text{and} \quad d \leq f(k), \tag{5}$$

where k and d denote respectively capital available for production and dividends distributed to shareholders at time 1. The price of a share q is known by the firm to take the form,

$$q = \frac{d}{\theta}, \tag{6}$$

where θ represents the opportunity cost of investing in capital for the firm. The value of θ is determined by the consumption discount factor of the consumers that are the marginal investors in the firm's stocks, and it is taken as given by the firm. Using (6) in (4), the optimal investment for the firm is determined by the familiar condition equating the marginal product of capital to its opportunity cost, formally,

$$f'(k) = \theta. \tag{7}$$

Therefore, under the firm's optimal investment strategy

$$q = \frac{f(k)}{f'(k)}, \tag{8}$$

which shows that investment, k, is positively related to the price of a share.

As indicated in Section 2 firms are indexed by $g \in [0,1]$, which represents a publicly observable measure of sustainability of their production activities. Notation-wise, we will denote capital, cost of capital, and price of a share for a firm g by $k(g)$, $\theta(g)$, and $q(g)$, respectively. We turn next to the household's problem.

Households, of types $i = A, B$, live for two periods, $t = 0, 1$. The initial portfolio holdings of household A in firm g are denoted by $z_0(g)$. Shares are traded in period 0 at the unitary price $q(g)$, and $z(g)$ denote the shares carried into the next period by household A. Throughout the analysis, we normalize the total number of outstanding shares of a firm to 1, so the initial portfolio endowment of household B is $1 - z_0(g)$, and market clearing implies that the number of firm g's shares that household B carries into period 1 is $1 - z(g)$.

Household A intertemporal preferences are specified as

$$\ln c_0^A + \beta \ln c_1^A + \mathcal{V}(\mathcal{G}), \tag{9}$$

where $\beta \in (0,1)$, and sustainable wealth $\mathcal{G}$ is defined as in (1). For household B, the preferences are simply

$$\ln c_0^B + \beta \ln c_1^B. \tag{10}$$

Note that both households have the same degree of patience β with respect to consumption utility, so that absent any additional motive, they would choose identical consumption growth between the two periods. The budget constraints for household A are

$$c_0^A = \int_0^1 \Big[z_0(g)d_0(g) - q(g)(z(g) - z_0(g))\Big]\phi(g)dg, \quad \text{and,} \quad c_1^A = \int_0^1 d(g)z(g)\phi(g)dg. \tag{11}$$

At time $t = 0$, household A receives dividend payments according to their initial portfolio holdings and then uses the funds to adjust their portfolio or to consume. At time $t = 1$, dividends are paid, and the total income is consumed. Throughout the analysis, we assume that households have a zero-borrowing constraint. We also assume that stocks cannot be shorted, which corresponds to imposing

$$z(g) \in [0,1] \quad \text{for all} \quad g \in [0,1]. \tag{12}$$

We discuss the implications of relaxing these assumptions in Section 3.5.

3.2 Optimal Portfolio Optimal portfolio allocation for household A requires that for each g,

$$\frac{c_1^A}{c_0^A} \geq \beta \frac{d(g)}{q(g)} + \mathcal{V}'(\mathcal{G}) v(g) c_1^A, \tag{13}$$

which holds with equality whenever $z(g) > 0$. Intuitively, for the stocks held in positive amounts, the marginal returns across all stocks should be equated and equal to consumption growth for the household. The right-hand side of (13) shows that the return consists of the sum of the pecuniary return – the dividend-yield $d(g)/q(g)$ appropriately discounted – and the non-pecuniary return measured in marginal utility of consumption, $\mathcal{V}'(\mathcal{G})v(g)c_1^A$. The non-pecuniary return can be positive or negative, depending on the sign of $v(g)$.

For the stocks that are not held in the household's portfolio, which corresponds to $z(g) = 0$, condition (13) implies that their total marginal return (pecuniary and non-pecuniary) is lower than those held in positive amounts.

For household B, the corresponding optimal portfolio allocation requires that for each g,

$$\frac{c_1^B}{c_0^B} \geq \beta \frac{d(g)}{q(g)}, \tag{14}$$

which now holds with equality whenever $z(g) < 1$, and it holds with inequality when $z(g) = 1$ (i.e. all stocks of firm g are held by household A).

In general equilibrium, stocks of all firms must be held by households, and stock prices and savings demand by households will have to be consistent with equations (13)-(14) for all $g \in [0,1]$. In turn, the equilibrium dividend yields impact the cost of capital for firms, and thus their capital allocation, since

$$\frac{d(g)}{q(g)} = \theta(g) = f'(k(g)). \tag{15}$$

We are ultimately interested in under what conditions and to what extent the presence of non-pecuniary motives in household's A preferences impacts capital allocation in equilibrium.

3.3 Equilibrium Let us first established a benchmark equilibrium for the case when $v(g) = 0$ for all $g \in [0, 1]$ for all investors, so absent any non-pecuniary motives for investment. In this case, capital allocated to each firm corresponds to

$$\hat{k} = \frac{\alpha\beta}{1 + \alpha\beta} \int_0^1 f\big(k_0(g)\big)\phi(g)dg. \tag{16}$$

Because all firms are symmetric, the portfolio allocation is indeterminate – that is, any portfolio holding is optimal – as long as the total value of the portfolio is equal to the desired savings. Aggregate output $\hat{y}$ is

$$\hat{y} = \int_0^1 f\big(\hat{k}\big)\phi(g)dg, \tag{17}$$

which represents the benchmark against which we will evaluate the scale impact of sustainable investing.

Let us now consider the case of advocate investors holding preferences with $v(g) \neq 0$ for some g. Differently from the benchmark case, when non-pecuniary motives are present, in equilibrium the holdings of stocks obeys an ordering dictated by the function $v(g)$. Specifically, an equilibrium is characterized by a threshold g^* such that for all values of $g \geq g^*$ household A will hold the entirety of the stocks in those firms, that is $z(g) = 1$, and for $g < g^*$ it will hold no stocks in those firms, that is $z(g) = 0$.

Under the g^* equilibrium, equations (13)-(14) suggest that for $g > g^* > \tilde{g}$ where g and $\tilde{g}$ are arbitrary values of the sustainability index, it must be that

$$\frac{d(g)}{q(g)} < \frac{d(\tilde{g})}{q(\tilde{g})} \implies \theta(g) < \theta(\tilde{g}) \implies f'(k(g)) < f'(k(\tilde{g})) \implies k(g) > k(\tilde{g}). \tag{18}$$

Intuitively, when holding a stock provides a non-pecuniary return, the investor is willing to hold the stock at a lower pecuniary return. If the investor happens to be the marginal investor in the stock (when $g > g^*$), the lower pecuniary return holds in equilibrium, lowering the cost of capital, and increasing investment. If the investors are not the marginal investor (when $g < g^*$), the pecuniary return is unaffected in equilibrium. Capital allocation is thus tilted towards firms with sustainability score higher than $g > g^*$.

An important insight from the general equilibrium of this simple model is that the observation of flows into stocks with higher sustainability scores does not necessarily imply a lower cost of capital for firms, and thus an impact on capital allocation. The impact happens only for the firms with $g > g^*$, where g^* is endogenously determined and dependent on parameters of the model, such as the initial wealth of advocate investors relative to benchmark investors, and the strength of their non-pecuniary motives.

Equation (18) provides information about the relative allocation of capital across firms with different g's, an effect that we will refer to as *composition impact,* but not about the impact on total capital, and thus total output in equilibrium. The re-allocation in (18) can in principle take place both when the overall capital is higher compared to the benchmark equilibrium (17). There is thus a second impact of sustainable investing that we want to investigate, which we will refer to as *scale impact*.

Furthermore, depending on the value of g^*, the composition effect might also be very limited or absent at all. Suppose that $g^* \approx 1$, which might happen when the wealth of advocate investors is small relative to the stock market capitalization of firms in a neighborhood of g^*. Equations (13)-(14) imply that the dividend yields must be the same across all firms with $g < g^* \approx 1$, so advocate investors have very limited impact on capital allocation.

3.4 Example: One Clean and One Dirty Firm

In the rest of this section, we investigate scale and composition impacts more systematically in the context of a model specification with two firms: a clean firm with sustainability score g, and a dirty firm with score $\tilde{g}$, with $g > \tilde{g}$. For the clean firm $v(g) = v > 0$, while for the dirty firm $v(\tilde{g}) = -\tilde{v} < 0$. In addition, we specify $\mathcal{V}'(\mathcal{G}) = 1$ and $f(k) = k^\alpha$, $\alpha \in (0, 1)$, to keep the algebra simple. None of our results crucially depend on these assumptions.[10]

Because the utility is logarithmic, the optimal saving strategy for households, absent any non-pecuniary motives, corresponds to the fraction $\frac{\beta}{1+\beta}$ of their initial wealth. For the benchmark investor this remains true even when advocate investors are present, so for Household B the saving

[10] We are implicitly assuming that the distribution of firms is degenerate so that, the mass of the clean firm and the dirty firm are both equal to 1/2. We have also dispensed with the existence of a neutral firm with $v(g_n) = 0$. Both assumptions have no consequences for the qualitative results in the two-period economy.

strategy can be always written as

$$q(g)(1 - z(g)) + q(\tilde{g})(1 - z(\tilde{g}) = \frac{\beta}{1+\beta}\Big[\big(1 - z_0(g)\big)V(g) + \big(1 - z_0(\tilde{g})\big)V(\tilde{g})\Big]. \tag{19}$$

When non-pecuniary motives are present, the saving strategy is modified to take into account the non-pecuniary marginal return of investing into the clean or the dirty firm. Three cases are possible in equilibrium, depending on the portfolio holdings of the advocate investor.

Case 1: $g^* = g$ with $z(g) < 1, z(\tilde{g}) = 0$. The saving function in this case is

$$q(g)z(g) = \frac{\beta + vq(g)z(g)}{1 + \beta + vq(g)z(g)}\Big[z_0(g)V(g) + z_0(\tilde{g})V(\tilde{g})\Big]. \tag{20}$$

The presence of non-pecuniary motives to hold the clean firm, $v > 0$, tilts the saving function upward, so a larger fraction of initial wealth is saved. Intuitively, the advocate investors are willing to postpone consumption in order to accumulate more wealth held in the form of stocks in the clean firm. In equilibrium, the return on savings must fall to accommodate the higher demand, which results in a lower dividend yield, or lower $\theta(g)$, and thus higher capital accumulation. Because the benchmark investor is the marginal investor in the clean firm in Case 1, it follows that $\theta(g) = \theta(\tilde{g})$, which implies the same capital allocation across the two firms,

$$k(g) = k(\tilde{g}). \tag{21}$$

Intuitively, the lower return on savings impacts the dividend yield of the dirty firm and thus capital accumulation in the dirty firm. Because of general equilibrium, both capital in the clean *and* dirty firm increase! In Case 1 we have a positive scale effect, but no composition effect.

Case 1 holds as long as the benchmark investors is the marginal investor, that is, as long as $z(g) < 1$, which happens when the initial wealth of the advocate investor is low or the non-pecuniary motive v is small. When either one of those gets larger, Case 2 applies.

Case 2: $g^* = g$ with $z(g) = 1, z(\tilde{g}) = 0$. In this case, the saving function is still equal to (20), but with $z(g) = 1$. The advocate investor is the marginal investor, so the dividend yield for clean stocks can be different from the dividend yield for dirty stocks. The advocate investor is willing to hold only clean stocks as long as their dividend yield is too low compared to that of the dirty stocks. That corresponds to a bound on the difference between capital allocated across the two firms, and can be expressed as

$$1 < \frac{k(g)}{k(\tilde{g})} \leq \left(1 + (v + \tilde{v})\frac{k(g)}{\alpha\beta}\right)^{\frac{1}{1-\alpha}}. \tag{22}$$

When the advocate investor is the marginal investor in the clean firm, there is a composition impact on capital allocation in favor of capital in the clean firm. The scale impact remains positive (more on this below). Note that Case 2 applies the larger is the non-pecuniary motive for investing in the clean firm, v, as well as the non-pecuniary motive for not investing in the dirty firm, $\tilde{v}$. Intuitively, both the positive utility from the clean holdings, and avoiding the negative utility from dirty holdings, keep the advocate investor from holding dirty stocks, thus lowering the required dividend yield on the clean stocks.

Case 3: $g^* = \tilde{g}$ with $z(g) = 1, z(\tilde{g}) > 0$. Here the advocate investor is the marginal investors for both the clean and the dirty firm. This happens when the initial wealth of the advocate investor is relatively high, and the investor is willing to hold dirty stocks as long as they offer an excess return over the clean stocks. The saving function in this case takes the form

$$q(g) + z(\tilde{g})q(\tilde{g}) = \frac{\beta - \sigma(\tilde{v})}{1 + \beta - \sigma(\tilde{v})}\left[z_0(g)V(g) + z_0(\tilde{g})V(\tilde{g})\right] + \frac{q\left(1 - \left(\frac{k(\tilde{g})}{k(g)}\right)^{1-\alpha}\right)}{1 + \beta - \sigma(\tilde{v})}, \tag{23}$$

where

$$\sigma(\tilde{v}) = \frac{\tilde{v}}{\tilde{v} + v}\beta\left(1 - \left(\frac{k(\tilde{g})}{k(g)}\right)^{1-\alpha}\right). \tag{24}$$

Note that Case 3 applies when the right inequality in (22) does not hold, so

$$\frac{k(g)}{k(\tilde{g})} > \left(1 + (v + \tilde{v})\frac{k(g)}{\alpha\beta}\right)^{\frac{1}{1-\alpha}}, \tag{25}$$

which also implies $\sigma(\tilde{v}) \geq 0$. The negative non-pecuniary return of holding dirty stocks has two competing effects on the saving strategy (23). On the one hand, the incentive to save is reduced because the investor wants to avoid holding dirty stocks, but it is still forced to do so in equilibrium. This is the first term on the right-hand side of (23). On the other hand, the capital allocation in favor of the clean stock increases its stock price q, and thus requires additional saving to be held the larger is the difference between $k(g)$ and $k(\tilde{g})$. This effect is represented by the second term on the right side of (23). Hence, in Case 3 the composition effect is largest, while the scale effect is ambiguous.

Figure 3 reports the numerical solution of the equilibrium in the two-firms' example. The standard parameters of the model are set at $\alpha = 0.33$, $\beta = 0.9$, $k_0(g) = k_0(\tilde{g}) = 1$. The left panels report the share of output produced by the clean firm (green area), and the dirty firm (gray area), plotted with respect to the fraction of the initial wealth held by the advocate investors, household A. Throughout the numerical example, the initial holdings of stocks are kept equal across clean and dirty firms, i.e. $z_0(g) = \tilde{z}_0(g)$, so that the fraction of initial wealth does not change because of the change of stock prices in equilibrium.[11] The right panels report the ratio of total output produced in the equilibrium with advocate investors and the total output under the benchmark symmetric capital allocation in (17), plotted against the fraction of the initial wealth held by advocate investors.

Panels (a-i)-(a-ii) consider the case of $v = 1$ and $\tilde{v} = 0$, so advocate investors derive non-pecuniary utility from clean stocks, but no negative utility from dirty stocks. Panel (a-i) shows that for a fraction of initial wealth below 43% of market capitalization, Case 1 applies. Advocate investors are not the marginal investors in the clean stocks, and so there is no composition effect: output is produced equally by the clean and dirty firm. However, the economy experiences a positive scale effect, which is visible in (a-ii): total output is higher than the benchmark even when capital is equally allocated. The reason is that advocate investors have a high saving demand because they are willing to sacrifice current consumption to hold clean stocks, and this puts downward pressure on the required pecuniary return on savings, θg. Because the marginal investor is the benchmark household, $\theta g = \theta \tilde{g}$, so the downward pressure on savings return affects the cost of capital of both firms, hence the higher investment and output.

[11] The fraction of initial wealth by advocate investors is defined as

$$\frac{z_0(g)V(g) + z_0(\tilde{g})V(\tilde{g})}{V(g) + V(\tilde{g})}$$

which is equal to $z_0(g)$ when $z_0(g) = z_0(\tilde{g})$. Intuitively, if the initial holdings of stocks are symmetric across agents, any impact on stock prices affects the value of wealth symmetrically, and thus leaves the fraction of wealth held initially across household unaltered. The variation on the horizontal axes in Figure 3 thus corresponds to changes in $z_0(g)$.

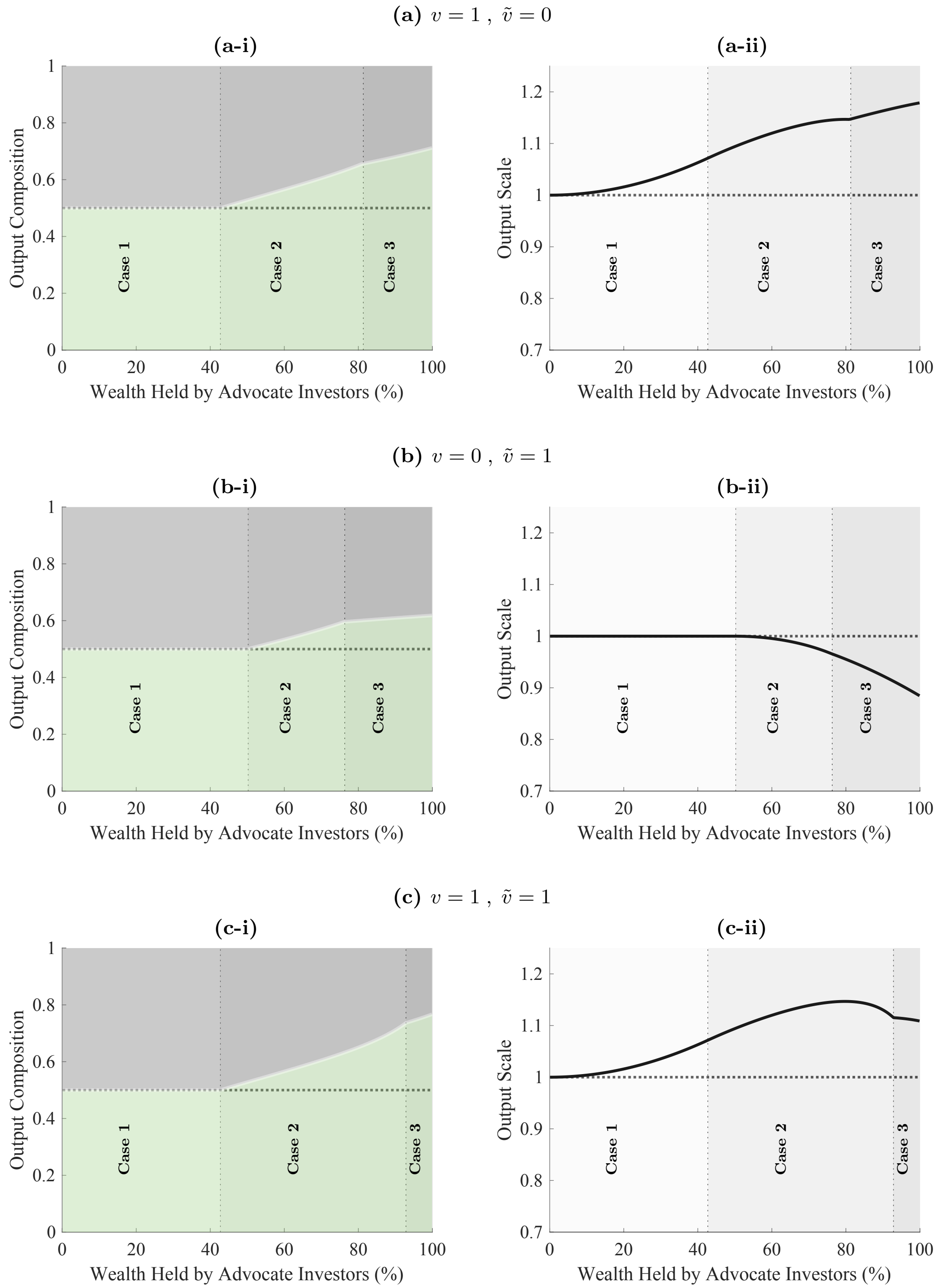

Figure 3: Composition and Scale Impact for the Clean and Dirty Firm: Examples

Figure 3 reports the composition impact (left panels), and scale impact (right panels) on output in the equilibrium of Cases 1-3, when $\alpha = 0.33$, $\beta = 0.9$, and $k_0 = \tilde{k}_0 = 1$. The horizontal axis measure the percentage of initial wealth held by Household A (Advocate Investors) at time $t = 0$. The sustainable wealth parameters are set at $v = 1$, $\tilde{v} = 0$ for (a-i)-(a-ii); $v = 0$, $\tilde{v} = 1$ for (b-i)-(b-ii), and $v = \tilde{v} = 1$ for (c-i)-(c-ii). For the composition effect, the gray area represent the fraction of output produced by the dirty firm, the green are the fraction produce by the clean firm.

When the initial wealth is above 43%, we are in Case 2, and the advocate investor is now the marginal investor. The cost of capital for the clean firm is lower than for the dirty firm, $\theta g < \theta \tilde{g}$, and so capital is disproportionately allocated to the clean firm, which is visible from the green area in Case 2 of the panel (a-ii). The total scale effect is still positive, and it is primarily driven by the higher capital in the clean firms. Finally, when the initial wealth is above 83%, we are in Case 3, and the advocate investor is the marginal investor in both firms. Here the difference in the cost of capital is the highest, and the higher the initial wealth, the higher the price the advocate investor is willing to pay for the clean stock. The increase in capital allocation here is mostly driven by the decrease in the marginal utility of consumption in period 1, which increases the non-pecuniary benefits of holding clean stocks measured in units of marginal utility.

Panels (b-i)-(b-ii) consider the case of $v = 0$, and $\tilde{v} = 1$, so advocate investors do not derive non-pecuniary utility from clean stocks, but do perceive negative utility from dirty stocks. Looking at (b-i), we see that the fraction of wealth required in this case to transition from Case 1 to Case 2, and see a composition effect, is higher than before, now at 50%. The reason for this comes from the saving function (20): when $v = 0$, the advocate investor saves a constant fraction $\beta/(1+\beta)$ of their initial wealth, which makes it more difficult to purchase all the clean stocks unless the initial wealth is higher. A further implication of the unaffected saving function is that the saving demand effect we have seen in (a-i)-(a-ii) is not present here, which means that there is no scale effect in Case 1, as visible in (b-ii). In Case 2, we see a positive composition effect towards the clean firm, as expected, but this corresponds to a negative scale effect visible in (b-ii)! The reason is that the saving demand is unchanged, but more capital is being allocated to the clean firm, so less capital is allocated to the dirty firm. Because of the decreasing marginal product of capital, the overall output decreases. The same mechanism also explains the increasing composition effect in Case 3, as well as the negative scale effect. In fact, in Case 3 the scale effect is quite large because of the impact of $k(\tilde{g})/k(g)$ on $\sigma(\tilde{v})$, and thus on the saving function (23).

Finally, panels (c-i)-(c-ii) consider the case of $v = \tilde{v} = 1$. First, the fraction of initial wealth for Case 1 to apply is the same as for the panels (a-i)-(a-ii), confirming that it depends on v, and not on $\tilde{v}$. So the emergence of composition impact depends on the positive non-pecuniary return from clean stocks, not on the dislike of dirty stocks. Second, the extent of Case 2 is larger than both previous set of panels. The reason is that the advocate investors will resist purchasing dirty stocks for a larger spread between the pecuniary returns on clean and dirty firms. This, in turn, increases the composition effect, which reaches the highest level in (c-i), compared to (a-i) and (b-i). Hence, everything else equal, a stronger composition effect is possible when non-pecuniary preferences for holding stocks are more extreme. However, the impact on the scale is non-monotone, as shown in (c-ii). The stronger incentive to allocate more capital to the clean firm, is not accompanied by an equally stronger savings demand by advocate investors. The savings demand is tempered by the possibility of having to hold the savings in dirty stocks. Hence, for a fraction of initial wealth higher than 80%, the allocation of capital to the clean firm comes at the expense of lower capital allocated to the dirty firm in such a proportion that overall output declines. The same pattern results in Case 3 of (c-ii). Hence, the presence of the negative non-pecuniary motive represented by $\tilde{v}$ acts as a drag on savings demand, as it does in panels (b-i)-(b-ii), thereby resulting in a non-monotone scale effect.

3.5 Discussion We use the results in the two firms example to discuss three important aspects of our model: borrowing constraints, arbitrage opportunities for firms, and disinvestment in so-called "sin" stocks.

3.5.1 The Role of Borrowing Constraints In our model, we have assumed that households cannot borrow from one another and cannot take short positions on stocks. Both assumptions are important for the existence of Case 1 because they prevent household A from holding all the stocks in the clean firm right away. Household A would be willing to borrow from Household B at the interest rate $R = \theta(\tilde{g})$ and purchase all the stocks in the clean firm. Indeed, household A would be willing to do so until the difference between the interest rate on borrowing and the pecuniary return on clean stocks equals the difference between the marginal product of capital at the boundary of Case 2 and Case 3! Hence, with a non-binding borrowing constraint, only Case 3 would emerge in equilibrium. With ad-hoc borrowing constraints, both Case 1 and 2 are still be part of the equilibrium. Our analysis assumes the strictest possible borrowing constraint by setting the borrowing limit to zero, but it can be generalized to less stringent constraints. The same argument applies to the short-sale constraints.

3.5.2 Arbitrage Opportunities for the Clean Firm Under the assumptions of our model, firms are restricted on the financial securities they can hold and issue. The implication is that there are arbitrage opportunities that remain unexploited. Let us take the equilibrium in Case 2 for example. The clean firm retains earning at the marginal cost, $\theta(g) < \theta(\tilde{g})$. The clean firm could in principle lend to the dirty firm at some interest rate $R \in [\theta(g), \theta(\tilde{g})]$ and thus reduce the cost of capital for the dirty firm. The equilibrium in this case would require $R = \theta(g) = \theta(\tilde{g})$, which implies that capital is equally distributed across firms, so the composition effect would not occur. Interestingly, the scale effect would still occur and extend to Case 2 as well, because of the saving strategy of the advocate investor.

The extent of the composition effect thus depends on the extent of the limits to set up financial securities across firms that face different cost of capital. The example also indicates the intricacies of establishing, on the side of the advocate investors, a measure of sustainable wealth that is truly reliable. In the arbitrate example above, the clean firm would hold debt assets issued by the dirty firm, but still be considered as sustainable as before by the advocate investors. If that were not the case, the clean firm would no longer have access to the lower cost of capital $\theta(g)$. Hence, the appropriate measure of sustainable wealth should evaluate the sustainability of both the productive and financial assets of a firm, and adjust the sustainability score accordingly. In the example above, this could mean that as soon as the clean firm lends to the dirty firm, it is scored at $\tilde{g}$ and thus loses the status of clean firm, and the access to cheaper capital. In reality, the indirect holdings of claims on firms' output by other firms is exceptionally complex, but this simple example suggests that closer scrutiny to the sustainability of productive activities behind financial assets is an important requirement for sustainability scores to translate into composition effect on capital allocation.

3.5.3 Disinvestment in "Sin" Stocks A question that we can address in our simple model is the impact on capital allocation of advocate investors choosing their portfolio holdings only considering negative non-pecuniary utility coming from dirty stocks. This would capture the case of disinvestment in "sin" stocks such as the tobacco or arms industries. This situation corresponds to $v = 0$, $\tilde{v} = 1$ and it is shown in panels (b-i)-(b-ii) in Figure 3. We first note that the saving function (20) reverts back to the standard log-case, a fraction $\frac{\beta}{1+\beta}$ of initial wealth. Summing (19) and (20) under $v = 0$ one obtains

$$q(g) + q(\tilde{g}) = V(g) + V(\tilde{g}). \tag{26}$$

Using (4) and (6), this corresponds to

$$k(g) = k(\tilde{g}) = \frac{1}{2}\Big[k_0(g)^\alpha + k_0(\tilde{g})^\alpha\Big], \tag{27}$$

which is the two-firm version of (16). Hence, for low values of advocate investors' initial wealth, preferences towards disinvestment in "sin" stocks have nor scale neither composition impact on capital allocation! Intuitively, the incentive to disinvest does not affect the overall saving strategy of the advocate investor when their wealth is all invested in clean stocks, but the benchmark investor is still the marginal investor. In other words, the scale effect we argued in Case 1 only happens when $v > 0$. This asymmetric is in part the consequence of the borrowing constraint (or short sale constraint) we have imposed on the problem.

4 Full Dynamic Model

We now extend the analysis to an infinite-horizon model to investigate whether the scale and composition effects uncovered above emerge in equilibrium, and whether they are temporary or permanent features of the capital allocation in the presence of sustainable investors. The details of all the derivations presented below can be found in Appendix A.2.

4.1 Firms We model the dynamic infinite-horizon problem of firms following Brock and Turnovsky (1981) and Turnovsky (1990). As in the two-period model, the firm owns capital directly and chooses an investment strategy that maximizes its total stock market value. The firm produces output using the technology f, and capital depreciates at the instantaneous rate $\delta > 0$ when used in production. Investment in capital is financed by retained earnings, and the remaining revenues are distributed as dividends, so that

$$d(g,t) = f(k(g,t)) - \dot{k}(g,t) - \delta k(g,t). \tag{28}$$

Let $\theta(g,t)$ denote the instantaneous return the firm's stock should provide in order to be held by households. The problem for the clean firm is to maximize

$$\int_0^\infty e^{-\int_0^t \theta(g,\tau)d\tau} d(g,t)dt, \tag{29}$$

by choosing a path for $d(t)$ that satisfies (28).

Letting $q(t)$ denote the market price of a share of the clean firm, because the outstanding number of stocks is normalized to 1, the total stock market value of the clean firm corresponds to $q(t)$. The instantaneous stock return consists of the divided flow and the capital gain, formally

$$\theta(g,t) = \frac{d(g,t)}{q(g,t)} + \frac{\dot{q}(g,t)}{q(g,t)}. \tag{30}$$

The solution of this differential equation, under $\theta(g,t) > 0$, and with boundary condition $\lim_{t\to\infty} q(g,t) < \infty$, is (29). Intuitively, the clean firm knows that the stock price must obey condition (30), and so it chooses a dividend path that implies a rate of capital accumulation that maximizes the stock price resulting from (30). The optimal strategy corresponds to the condition,[12]

$$f'(k(g,t)) = \delta + \theta(g,t).$$

The marginal return on capital net of depreciation is equated to the return on stocks required by the marginal investor in the clean stocks.

4.2 Households

The objective function for advocate households is specified as

$$\int_0^\infty e^{-\rho t} \left[\ln c^A(t) + \mathcal{V}\big(\mathcal{G}(t)\big)\ \right] dt, \tag{31}$$

where $\rho > 0$ is the subjective discount rate. Sustainable wealth $\mathcal{G}(t)$ is now time-varying because the portfolio holdings are time-varying, so that

$$\mathcal{G}(t) = \int_0^1 v(g)z(g,t)q(g,t)\phi(g)dg. \tag{32}$$

[12] See the Appendix A.2 for the derivation.

Total wealth for household A at time t is defined as

$$a^A(t) = \int_0^1 z(g,t)q(g,t)\phi(g)dg, \tag{33}$$

which evolves according to the differential equation

$$\dot{a}^A(t) = \int_0^1 \left[d(g,t) + \dot{q}(g,t)\right] z(g,t)\phi(g)dg - c^A(t). \tag{34}$$

4.3 Equilibrium As it was the case in the two-period model, the equilibrium is characterized by a threshold sustainability score, with the difference that it is now time-varying. More precisely, the dynamic equilibrium consists of a $g^*(t) \in [0,1]$ such that for $g \geq g^*(t)$ household A holds the entirety of stocks in those firms, which means $z(g,t) = 1$, while it holds zero stocks in all the remaining firms. Optimal portfolio allocation and the time-varying threshold equilibrium imply

$$\frac{\dot{c}^A}{c^A} = \theta(g^*) + v(g^*)c^A - \rho \tag{35}$$

and

$$\frac{\dot{c}^B}{c^B} = \theta(g^*) - \rho, \tag{36}$$

where we have suppressed time-dependence for notational convenience. The difference between consumption growth for the two households is proportional to the non-pecuniary return of the threshold firm, $v(g^*)$. When $g^* > g_n$, $v(g^*) > 0$, so consumption growth for household A is larger than for B, which means that the household is saving relatively more and thus expanding its portfolio. When $g^* < g_n$, on the other hand, $v(g^*) < 0$, and consumption growth for household A is now smaller than for B, suggesting that A is saving less, thus reducing its portfolio. This intuition is confirmed by the law of motion for g^* which can be expressed as

$$\dot{g}^* = \frac{c^A - \int_{g*}^1 d(g)\phi(g)dg}{q(g^*)\phi(g^*)}. \tag{37}$$

The numerator consists of the difference between household A's consumption and income at time t, the latter consisting of the sum of dividend payments from its portfolio at time t. If the difference is negative, it means that some of the income is used to increase savings, which is only possible by increasing the holdings of firms with a lower score than the ones already in the household's portfolio, so that $\dot{g}^* < 0$. If the difference is positive, to sustain the higher consumption the household is liquidating part of its portfolio starting from the firms with lower sustainability scores, and so $\dot{g}^* > 0$. Finally, for a given difference in consumption and income, the rate of change of the threshold g^* is lower the higher is the stock price of the threshold firm, $q(g^*)$, and the higher is the mass of firms at that price, $\phi(g^*)$.

The optimal portfolio conditions for household A imply that

$$\theta(g) - \theta(g^*) = -\big(v(g) - v(g^*)\big)c^A \quad \text{for} \quad g \geq g^*, \tag{38}$$

and those for household B imply

$$\theta(g) = \theta(g^*) \quad \text{for} \quad g < g^*. \tag{39}$$

Equations (38)-(39) outline the characterization of capital allocation in equilibrium that is analogue to Cases 1-3 in the two-firms example analyzed above. When equation (38) for a firm g, if g is above the neutral score g_n, capital allocation is tilted in its favor compared to the threshold firm g^*, while if g is below the neutral score the tilt is reversed. The relevant reference for evaluating overall impact is thus whether g^* is higher or lower than the neutral score g_n. When $g^* > g_n$, equation (38) implies that all the firms with score higher than g^* are positively impacted by advocate investors, so we should expect both a composition and a positive scale effect. However, when $g^* < g_n$, the "clean" firms, those with $g > g_n$ register a positive composition impact, but the "dirty" firms register a negative composition impact. The scale impact overall is ambiguous, and it depends on the location of g^* and the distribution of firms across sustainability scores.

The full dynamic model allows us to investigate how the equilibrium changes over time, and where the economy is headed in the long run. Towards that goal it is useful to compare the evolution of wealth across the two households, which are

$$\dot{a}^A = \theta(g^*)a^A - \big(1 + \mathcal{G} - v(g^*)a^A\big)c^A, \tag{40}$$

and

$$\dot{a}^B = \theta(g^*)a^B - c^B. \tag{41}$$

Both processes are driven by the benchmark return on wealth, $\theta(g^*)$, minus the part of the return that goes into flow consumption. When combined with (36), equation (41) implies that household B consume a constant fraction of its wealth, which is the familiar log-utility condition $c^B = \rho a^B$.

The difference between the two is represented by the term, $\mathcal{G} - v(g^*)a^A$, which represents the difference between the effective measure of sustainable wealth held by household A at t, $\mathcal{G}(t)$, and the lowest equivalent sustainability measure of that wealth, as if all stocks were of g^*, represented by $v(g^*)a^A$. Note that

$$\mathcal{G} - v(g^*)a^A = \int_{g^*}^{1} \big(v(g) - v(g^*)\big)q(g)\phi(g)dg, \tag{42}$$

so the value of this term is always positive. Equation (40) shows that household A behaves as if current consumption level is always more expensive in terms of sacrifice in wealth growth necessary to maintain it. At the same time, equation (36) implies higher consumption growth whenever $v(g^*) > 0$ (equivalently $g^* > g_n$), which corresponds to lower consumption levels early along the optimal path, and thus faster growth accumulation overall. Symmetrically, when $v(g^*) < 0$ (equivalently $g^* < g_n$), equation (36) implies lower consumption growth, and thus higher consumption levels early along the optimal path, and thus slower, or negative wealth accumulation. Taken all the equations together, the dynamic path can be summarized in the following proposition.

Proposition 1. *The dynamic path of the full dynamic model with advocate investors can be summarized as follows. When $g^*(t) > g_n$, household A accumulates wealth along the optimal path, which implies $\dot{g}^* < 0$. When $g^*(t) < g_n$ household A decumulates wealth along the optimal path, which implies $\dot{g}^* > 0$.*

Whether the initial threshold $g^*(0)$ is above or below the neutral level g_n depends on the initial wealth distribution across households. In particular, if household A begins with a level of wealth that is relatively low compared to B, then the path with $\dot{g}^* < 0$ applies. If household A initial wealth is relatively large, the path with $\dot{g}^* > 0$ applies. This implication has the same qualitative flavor of the equilibrium with two-firms, where, depending on the initial wealth level of A, one would see a gradual wealth accumulation, or a gradual wealth decumulation.

The natural question at this point is where the economy is headed in the long run, that is, what is the steady state of the dynamic system we just investigate. The following proposition states the result.

Proposition 2. *In the steady state of the equilibrium of the full dynamic model with advocate investors*

$$g^* = g_n \tag{43}$$

and capital allocation is determined by

$$f'(k_{ss}(g)) = \rho + \delta - v(g)\int_{g_n}^{1}\left[f(k_{ss}(h)) - \delta k_{ss}(h)\right]\phi(h)dh, \quad \textit{for} \quad g \geq g_n, \tag{44}$$

and

$$f'(k_{ss}(g)) = \rho + \delta, \tag{45}$$

otherwise.

Before commenting on the result, let us state that in the economy where only benchmark investors are present, capital in steady state is the same across all firms and determined by the condition

$$f'(\hat{k}) = \rho + \delta \quad \text{ for all } \quad g \in [0,1]. \tag{46}$$

In the steady state of an economy with advocate investors things are different. Advocate investors hold a portfolio that consists of all the firms that have a score above or equal to the neutral score g_n. The rest of the stocks are held by the benchmark investors.

For the firms held by the advocate investors, capital composition and scale is determined by equation (44). Note that the term under the integral is always positive as it coincides with total dividend income, and thus total consumption, of household A in steady state. It follows that capital allocated to "clean" firms is always higher than what would be the case in the benchmark economy. Interestingly, the extent of the scale of capital re-allocation depends on the distribution of firms above the neutral score g_n, that is $\phi(g)$ for $g > g_n$. The more the distribution is skewed towards firms that have a higher sustainability score, the more capital will be allocated to sustainable firms overall. Intuitively, capital allocation is a consequence of the saving of advocate investors in the long run. With more firms on the “clean” side of the sustainability distribution, advocate investors reduce consumption further early along the path to make sure enough capital flows to those firms, this is what their preferences imply, after all! Because the long run is a long time, household A is patient enough to eventually save so that capital scale is higher, the higher is the mass of sustainable firms.

For firms below the neutral score, long run capital allocation is the same as in the benchmark economy. In some respects, this is a surprising result, given that advocate investors harbor negative preferences for those firms, and we know from the two-period model that the scale effect can be negative when advocate investors really dislike dirty firms. However, both general equilibrium and long run dynamics operate to completely neutralize the impact of preferences for disinvestment on dirty firms. General equilibrium implies that such firms must be held by households, and thus continue to operate providing the return desired by their marginal investor. The long run dynamics implies that advocate investors gradually reduce their holding of dirty stocks, and if the equilibrium where $g^*(0) < g_n$ applies, they do so by reducing their saving, while benchmark investors maintain their saving. Eventually, the marginal investor in dirty firms is household B, and household's B long-run saving implies the allocation (45).

Proposition 2 also shows that aggregate capital and output in steady state are a function of the preferences of advocate investors. If one thinks of $v(g)$ as modifying the patience of households into a degree of “effective patience,” the result can be interpreted as the generalization of the familiar dependence of steady state capital on the subjective discount rate ρ. Thus, when $v(g) > 0$ household are willing to permanently substitute consumption for capital $k(g)$, hence the higher steady state capital. Our result also shows that for the dirty firms, i.e. those with $v(g) < 0$, capital remains unaffected in the steady state, even if household A is relatively more impatient and not willing to substitute consumption for capital in dirty firms. The reason is that the patience of household A does not matter for capital allocation in dirty firms, while the patience of household B now matters. Since household B has a higher degree of effective patience for capital in dirty firms, they will be the marginal investors determining the steady state capital allocation for the dirty firms.

4.4 Numerical Example Figure 4 reports the numerical solution of the steady state equilibrium of Proposition 2 under two specifications for the non-pecuniary preferences $v(g)$, reported in the top panel: one with a low neutral score, $g_n^{low} = 0.4$, and plotted in plain; and one with high neutral score, $g_n^{high} = 0.6$, and plotted in dashed. The low neutral score preferences correspond to advocate investors that are both less restrictive in the way they map sustainability scores into non-pecuniary returns, and more balanced across clean firms. This is captured by the plain line being positive for a larger fraction of firms, and plateauing quickly after a score of 0.6. The high neutral score preferences correspond to advocate investors that are more restrictive in mapping sustainability scores into non-pecuniary returns, and less balanced, in the sense that they assign large non-pecuniary return to the stocks that they consider clean. This is captured by the dashed line being larger than the plain line for scores above 0.7. For both computations we use the firms density corresponding to the observed employment size by score decile of Figure 2, panel (b), and plotted in the middle panel of Figure 4.

The bottom panel of Figure 4 reports the composition impact in steady state as a fraction of the steady state output in a benchmark economy without advocate investors. The plain line represents the output composition under the low neutral score preferences, while the dashed line is for the high neutral score preferences. Both are 1 for scores below the neutral score, and are larger than 1 and increasing in the sustainability score for higher values. The plain line corresponds to a more balanced composition compared to the dashed line, in part reflecting the similar shape of the underlying non-pecuniary preferences. Overall, the total scale impact on output is similar across the two cases: 1.13 for the plain, and 1.15 for the dashed.

Interestingly, the shape of the output composition in Figure 4 is the consequence of the combined effect of the shape of the preferences $v(g)$, and the empirical size distribution, $\phi(g)$. Figure 5 reports the output composition under the same preferences of the two cases of Figure 4, but using a uniform distribution for firms size, corresponding to 10% for each score decile. In this case, the composition impact is smaller in both cases, but the low neutral score monotonically dominates the composition of the high neutral score! The overall output scale impact is now 1.06 for the plain vs 1.03 for the dashed.

The explanation offers important insights on the mechanisms of the model. In the firms size distribution of Figure 4, 70% of firms, and thus most of the market value, is located above $g = 0.6$,

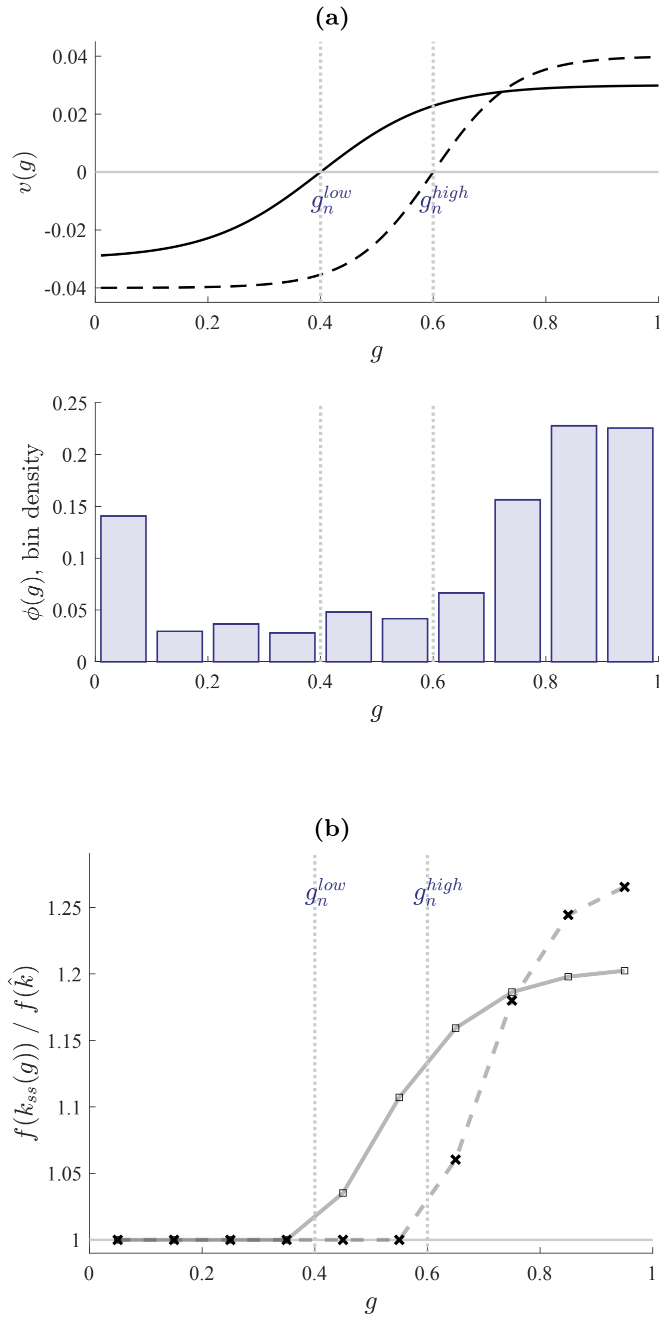

Figure 4: Output Composition Impact in Steady State (w. Empirical Density)

Figure 4 reports the steady state composition impact on output across firms (bottom panel) based on the observed density of firms across sustainability scores, $\phi(g)$, measured in terms of employment (middle panel), and two alternative preferences for non-pecuniary returns on stocks, $v(g)$ (upper panel). The low neutral score, $g_n^{low} = 0.4$, is plotted in plain; the high neutral score, $g_n^{high} = 0.6$, is plotted in dashed. Baseline parameters are set at $\alpha = .33$, and $\rho = 0.11$. The output scale impact is 1.13 for the plain case, and 1.15 for the dashed case.

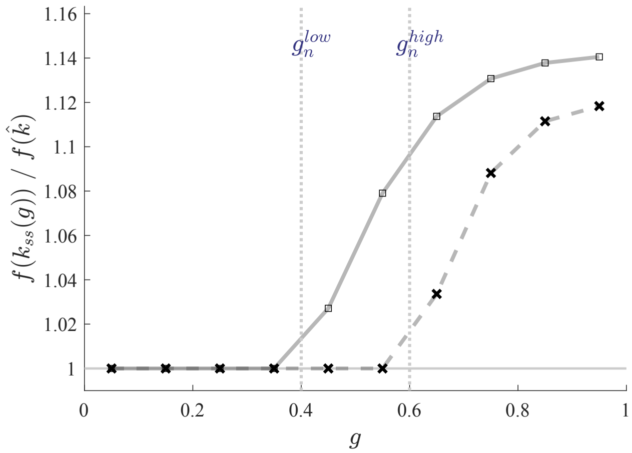

Figure 5: Output Composition Impact in Steady State (w. Uniform Density)

Figure 5 reports the the steady state composition impact on output for the two alternative preferences for non-pecuniary returns on stocks, $v(g)$, of panel (a) in Figure 4, under a uniform distribution of firms size. The output scale impact is 1.06 for the plain case, and 1.03 for the dashed case.

and only 10% of the firms are between $g = 0.4$ and $g = 0.6$. This means that the total saving in steady state for both preferences specifications is similar because investors have to hold a similar share of the market in both cases, and the composition mostly reflects the different shape of $v(g)$, for $g > g_n$. With a uniform distribution, 40% of the firms are above $g = 0.6$ and 20% are between $g = 0.4$ and $g = 0.6$, so to hold the market capitalization in the case of the low neutral score the advocate investors must accumulate more savings compared to the high neutral score case. This results in a higher capital allocation across all firms with $g > 0.4$, and hence a larger scale effect. Finally, because overall less saving has to be accumulated in the uniform distribution case, the scale effect is smaller compared to the empirical distribution case of Figure 4.

The numerical example highlights the subtle interaction between the shape of non-pecuniary preferences for advocate investors, their perception of a neutral score, and the distribution of firms across sustainability scores, in shaping both the composition and the scale impact of sustainable investing in general equilibrium.

5 Conclusion

In this paper we characterize under what conditions sustainable investing—financial investors who prefer (resp. dislike) holding assets that they believe have a positive (resp. negative) environmental/societal impact— alter the allocation of capital between clean and dirty firms in a dynamic general equilibrium framework. We show that advocate investors can have both a *scale* and a *composition* impact on capital allocation and aggregate output. We show how the value of total wealth of advocate investors relative to total market capitalization, the shape of their preferences for non-pecuniary returns of holding clean and dirty stocks, and the size of firms across sustainability score, interact in general equilibrium to determine the extent of the impact of sustainable investing. The model has rich predictions for the scale and composition effects, and suggests interesting venues for the role of policy to address important sources of environmental and societal externalities in the presence of sustainable investing. We explore such implications in ongoing work in progress.

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Appendix A Model Details

A.1 Solve Households' Optimization Problem

To be added.

A.2 Derive Firms' Optimal Investment Strategy

To be added.