IAERE Summer School 2022

Università di Urbino

The Macroeconomic Theory of Sustainability: An Introduction

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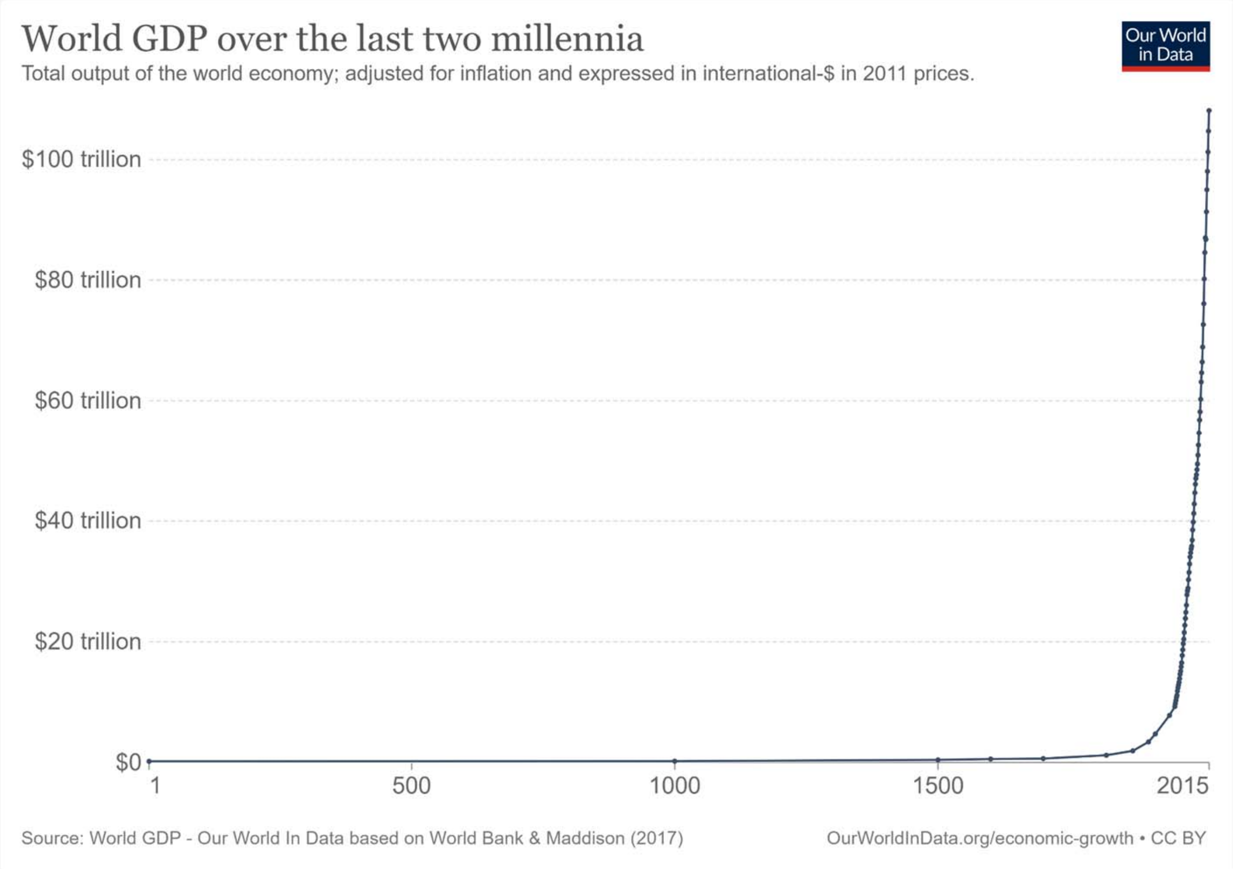
Lecture 1



Can existing patters of human activity safely and sensibly continue unaltered over the long term, or will such continuation lead to unacceptable consequences? (Heal, 1998)

The Sustainability Issue

"Sustainable development consist in meeting the needs of the present without compromising the ability of *future generations* to meet their own needs"

Bruntland Report (1987)

"If sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the *very long run*. It is very important to understand what that thing is. I think it has to be a *generalized capacity to produce economic well-being*."

Robert Solow (1993, pp 167-168)

Sustainability: Two Opposite(?) Paradigms

- **Strong Sustainability**: natural capital (environmental assets) is fundamentally not substitutable by other forms of human-made capital and it must be preserved for future generations

- **Weak Sustainability**: the well-being of future generations must be preserved; natural and human-made capitals are of interest in their role of producing goods and services contributing to the well-being.

Operational Notion of Sustainability (Heal, 1998)

Sustainability in practice requires one to

1. consider **long-run** consequences of current choices to account for **intergenerational equity**

2. accounting for constraints implied by **dynamics** of **natural resources**

3. recognize that **natural resources** may **directly contribute** to economic **well-being**

Overview of Lectures

Lecture 1:

i. review of dynamic decision-making

ii. efficiency and intergenerational equity

iii. macro approach to sustainability

Lecture 2:

i. sustainability criteria

ii. conservation of non-renewable and renewable resources

Lecture 3:

i. natural resources and reproducible capital

ii. sustainability and national accounting

A note on the methodology

- the three lectures are a selected introduction to the theory of sustainability

- focus is on stylized models with a “neo-classical” flavor

- models not to be taken literally, but used to discover insights that extend to more sophisticated/quantitative environments

- mathematics used as a language, will do my best to not turn it into a barrier

A note on the methodology (cont'ed)

- we will study models in which ecological dynamic processes are kept in the background

- the same models allow the integration of realistic (non-linear, chaotic, etc.) ecological dynamics

- finally: understanding is prioritized over coverage, please stop me if anything is unclear (I will share all the slides I prepared)

Textbook References

Martinet (2012): Economic Theory and Sustainable Development

Heal (1998): Valuing the Future

Pezzey and Toman (2002): The Economics of Sustainability

Asheim (2007): Justifying, Characterizing and Indicating Sustainability

Math:

Kamien and Schwartz (1991): Dynamic Optimization

Weitzman (2003): Income, Wealth, and the Maximum Principle

Review of Dynamic Decision-Making

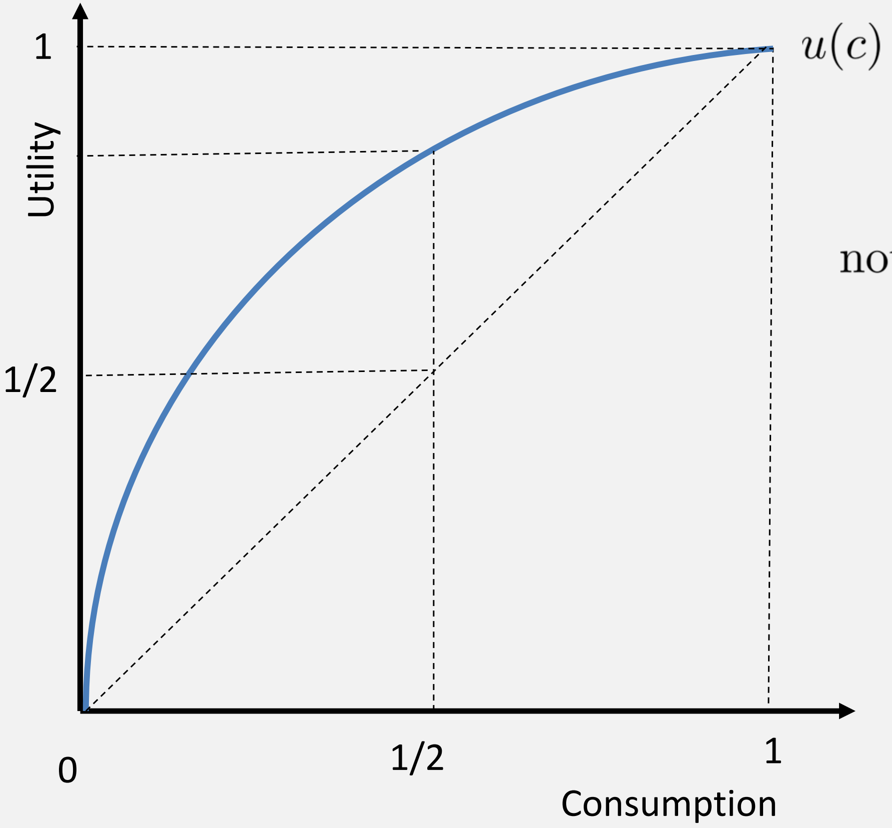
A Simple Dynamic Problem (1)

$$\max \quad u(c_1) + u(c_2), \quad \text{such that} \quad 1 - c_1 = c_2$$

$$\text{solution: } u'(c_1) = u'(1 - c_1) \implies c_1 = 1/2 = c_2$$

intepretation: allocation of resources across two subsequent generations

Aversion to Intergenerational Inequality



elasticity of marginal utility

$$\eta(c) = -\frac{cu''(c)}{u'(c)}$$

note: η favors **equality** across generations

$\eta(c) = \eta$: isoelastic utility

$$u(c) = \frac{c^{1-\eta}}{1-\eta}$$

$\eta = 0$: linear utility

$\eta \to \infty$: Leontief utility

A Simple Dynamic Problem (2)

$$\max \quad u(c_1) + u(c_2), \quad \text{such that} \quad (1 - c_1)(1 + \rho) = c_2$$

$$\text{solution: } u'(c_1) = (1 + \rho)u'(1 - c_1)$$

with iso-elastic utility

$$\frac{c_2}{c_1} = (1 + \rho)^{(1/\eta)}$$

note: $\rho > 0$ favors consumption of **future** generation

A Simple Dynamic Problem (3)

$$\max \quad u(c_1) + \frac{1}{1+\delta} u(c_2), \quad \text{s.t.} \quad (1 - c_1)(1+\rho) = c_2,$$

δ: discount rate $\qquad \frac{1}{1+\delta}$: discount factor

solution: $u'(c_1) = \left(\frac{1+\rho}{1+\delta}\right) u'(1 - c_1)$ **"Keynes-Ramsey" rule**

with iso-elastic utility

$$\frac{c_2}{c_1} = \left(\frac{1+\rho}{1+\delta}\right)^{(1/\eta)}$$

note: $\delta > 0$ favors consumption of **current** generation

Taking Stock

the prototypical intertemporal maximization problem features

- intergenerational inequality aversion (η)
- intergenerational equity (δ)
- intergenerational efficiency (ρ)

let us now model the very long run...

The Solow Model

- time is continuous and runs forever ($t \in [0, \infty)$)
- $K(t)$: stock of capital at time t
- $F(K(t))$: flow of output at time t (F is a "production function")
- $c(t)$: consumption at time t, it is $(1 - s)F(K(t))$

dynamic equation for capital (capital accumulation)

$$\dot{K}(t) = sF(K(t)) - \lambda K(t), \qquad \text{where} \quad \dot{K}(t) \equiv \frac{dK}{dt}$$

s: saving rate; λ: depreciation rate

The Solow Model: Steady State

the steady state is where the economy is headed in the **very long-run**

steady state: stock of capital $\hat{K}$ such that $\dot{K}(t) = 0$

$$\dot{K}(t) = sF(K(t)) - \lambda K(t)$$

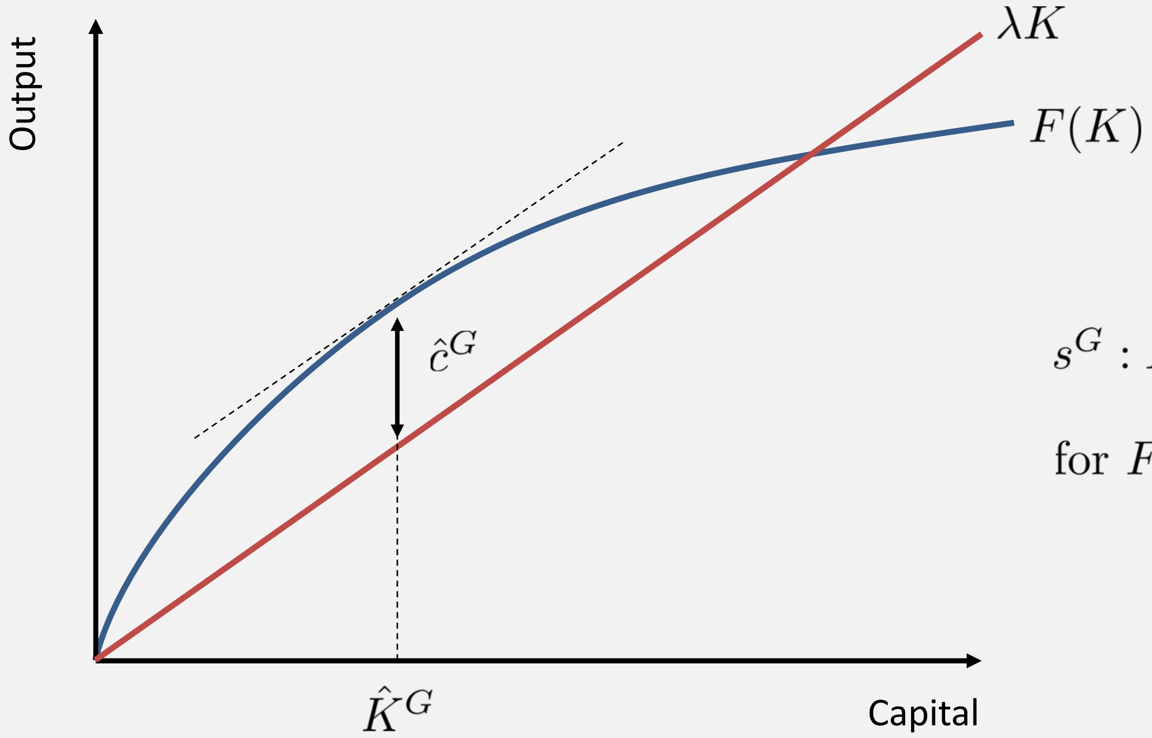
$$sF(\hat{K}) = \lambda \hat{K}$$

let $F(K) = K^\alpha$, then

$$\hat{K}(s) = \left(\frac{s}{\lambda}\right)^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad \hat{c}(s) = (1-s)\left(\frac{s}{\lambda}\right)^{\frac{\alpha}{1-\alpha}}$$

The Golden-Rule of Capital (Phelps, 1961)

What is the saving rate that maximizes consumption in steady state?



$$s^G : F'(\hat{K}(s)) = \lambda$$

for $F(K) = K^\alpha$, $s^G = \alpha$

Review of Cass-Koopmans Optimal Growth Model

$$\max_{c(\cdot)} \int_0^\infty u(c(t))e^{-\delta t}dt$$

$$\text{s.t.}: \dot{K}(t) = F(K(t)) - \lambda K(t) - c(t), \quad K(0) \text{ given}$$

- discount rate is δ, discount factor is $e^{-\delta t}$
- $\int_0^\infty$ "sums" utility across periods weighted by discount factor
- $F(K(t)) - c(t)$ was $(1-s)F(K(t))$ in Solow model, here s is chosen optimally

Review of Cass-Koopmans Optimal Growth Model

Solve problem using **Optimal Control**

Hamiltonian: $\mathcal{H}(c, K, p) \equiv u(c(t)) + p(t)\dot{K}(t)$

c: control variable $\rightarrow$ $\dfrac{\partial \mathcal{H}}{\partial c} = 0$

K: state variable $\rightarrow$ $\dfrac{\partial \mathcal{H}}{\partial K} = \delta p(t) - \dot{p}(t)$

p: co-state variable $\rightarrow$ $\dfrac{\partial \mathcal{H}}{\partial p} = \dot{K}(t)$

Review of Cass-Koopmans Optimal Growth Model

Hamiltonian: $\mathcal{H}(c, K, p) = u(c(t)) + p(t)\left(F(K(t)) - \lambda K(t) - c(t)\right)$

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \quad \rightarrow \quad u'(c(t)) = p(t)$$

$$\frac{\partial \mathcal{H}}{\partial K} = \delta p(t) - \dot{p}(t) \quad \rightarrow \quad \delta p(t) - \dot{p}(t) = p(t)\left(F'(K(t)) - \lambda\right)$$

$$\rightarrow \quad \frac{\dot{p}(t)}{p(t)} = \delta - \left(F'(K(t)) - \lambda\right)$$

$$\frac{\partial \mathcal{H}}{\partial p} = \dot{K}(t) \quad \rightarrow \quad \dot{K}(t) = F(K(t)) - \lambda K(t) - c(t)$$

Review of Cass-Koopmans Optimal Growth Model

- take time derivative of $u'(c(t)) = p(t)$

$$u''(c(t))\dot{c}(t) = \dot{p}(t)$$

- divide both sides by $u'(c(t)) = p(t)$

$$\frac{u''(c(t))}{u'(c(t))}\dot{c}(t) = \frac{\dot{p}(t)}{p(t)} \quad \rightarrow \quad -\eta\frac{\dot{c}(t)}{c(t)} = \frac{\dot{p}(t)}{p(t)}$$

- use expression for $\dot{p}(t)/p(t)$ from slide before and rearrange

$$\frac{\dot{c}(t)}{c(t)} = \frac{F'(K(t)) - \lambda - \delta}{\eta}$$

Review of Cass-Koopmans Optimal Growth Model

$$\frac{\dot{c}(t)}{c(t)} = \frac{F'(K(t)) - \lambda - \delta}{\eta}$$

$F'(K(t)) - \lambda$: saving return favors consumption of **future** generations ($\dot{c}(t) \uparrow$)

δ: discount rate favors consumption of **current** generations ($\dot{c}(t) \downarrow$)

η: intergenerational inequality aversion favors **equality** ($\dot{c}(t) \rightarrow 0$)

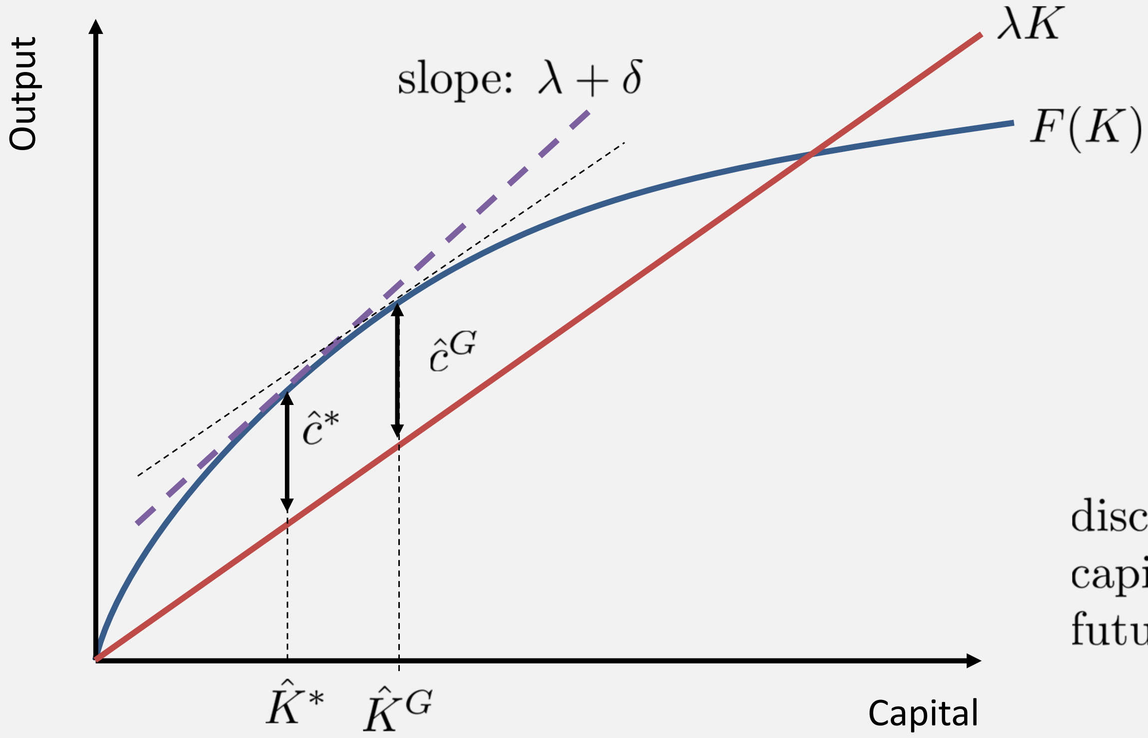
next: study the steady state

Review of Cass-Koopmans Optimal Growth Model

steady state: $\dot{c}(t) = 0$

$$\frac{\dot{c}(t)}{c(t)} = \frac{F'(K(t)) - \lambda - \delta}{\eta} = 0 \rightarrow F'(\hat{K}) = \lambda + \delta$$

Modified Golden-Rule of Capital



$$\hat{K}^G : F'(\hat{K}) = \lambda$$

$$\hat{K}^* : F'(\hat{K}) = \lambda + \delta$$

discounting $\delta > 0$ leads to lower capital accumulation for future generations!

Time Preferences and Social Discount Rate

What should be the discount rate used to evaluate **social optimality** of projects?

$$R(t) \equiv F'(K(t)) - \lambda$$

Optimal Growth model says

$$R(t) = \delta + \eta \frac{\dot{c}(t)}{c(t)}$$

Social discount rate depends on $\dot{c}(t)$ and intergenerational inequality aversion!

1) $\dot{c}(t) = 0$, then $R(t) = \delta$

2) $\dot{c}(t) > 0$, then $R(t) > \delta$

3) $\dot{c}(t) < 0$, then $R(t) < \delta$

note: even with $\delta = 0$, social discount rate can be positive or negative!

Let's Take Stock

classic intertemporal optimization has seeds of sustainability elements

- aversion to intergenerational inequality
- discounting and intergenerational equity
- capital accumulation for future generations (very long run)
- social discount rate

Macroeconomics Approach to Sustainability

1. **classical**: introduce additional constraints in standard optimal growth
   - model productive role and dynamics of natural resources explicitely
   - impose that consumption or utility are non-decreasing over time
2. **alternative**: embed sustainability value in criterion of optimality
   - specify criterion that trades off efficiency and intergenerational equity
   - evaluate dynamics under constraints due to natural resources (as in 1.)

Homework: The "Cake-Eating" Economy

Consider the problem of a social planner that has to decide how much of a cake of size $S(0)$ should be eaten in each instant to maximize the sum of discounted utility, formally:

$$\max_{c(.)} \int_0^\infty \frac{c(t)^{1-\eta}}{1-\eta} e^{-\delta t} dt$$

$$\dot{S}(t) = -c(t), \quad S(t) \geq 0 \text{ and } S(0) \text{ given}$$

a) write the Hamiltonian associated with the maximization problem

b) obtain the necessary conditions for an optimal solution

c) what is a key feature of the co-state variable along the optimal path?

d) characterized the optimal path for consumption $c(t)$