IAERE Summer School 2022
Università di Urbino

The Macroeconomic Theory of Sustainability: An Introduction

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Lecture 2

"The relation between economics and ethical principles is not linear but rather iterative. Examination of the implications of moral principles in particular models may lead to their revision. By applying ethical criteria to concrete economic models, we learn about their consequences, and this may change our views about their attractiveness."

Atkinson (2001)

"*The Strange Disappearence of Welfare Economics*"

Overview of Lectures

Lecture 1:

i. review of dynamic decision-making

ii. efficiency and intergenerational equity

iii. macro approach to sustainability

Lecture 2:

i. sustainability criteria

ii. conservation of non-renewable and renewable resources

Lecture 3:

i. natural resources and reproducible capital

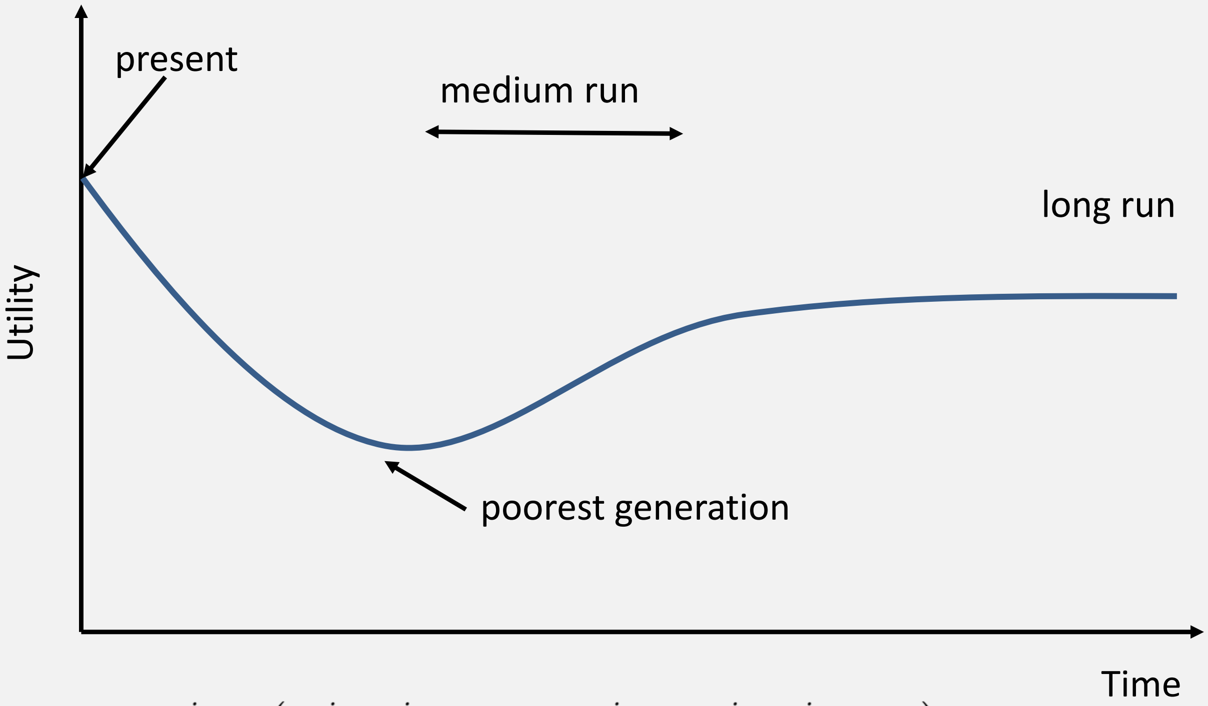
ii. sustainability and national accounting

Overview of Macroeconomic Approach

1. classical: introduce additional constraints in standard optimal growth

2. alternative: embed sustainability value in criterion of optimality

An Arbitrary Utility Stream



utility stream: $U^i \equiv (U_1^i, U_2^i, U_3, ..., U_{t-1}^i, U_t^i, U_{t+1}^i, ...)$

evaluation criterion: $V^i = SWF(U^i)$

Intertemporal (Intergenerational) Preferences

utility stream: $U^i \equiv (U^i_1, U^i_2, U_3, ..., U^i_{t-1}, U^i_t, U^i_{t+1}, ...)$

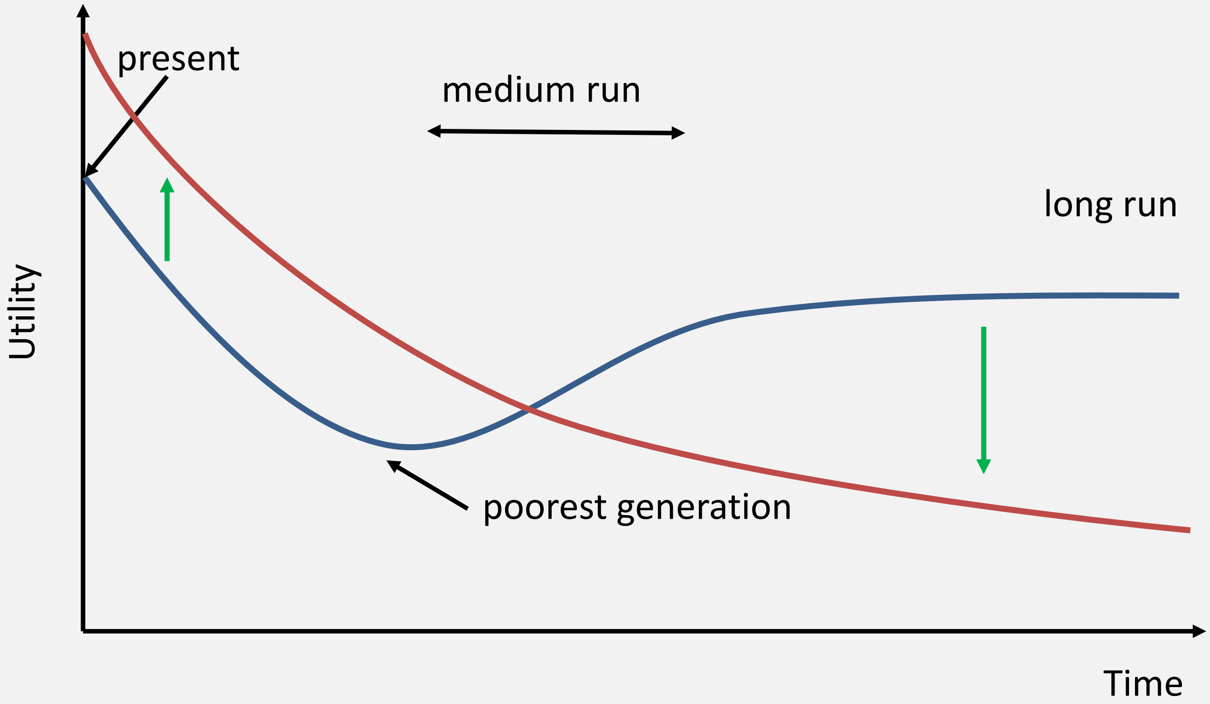
notation:

- $U^i \geq U^j$ if $U^i_t \geq U^j_t$ at any time t
- $U^i > U^j$ if $U^i_t \geq U^j_t$ at any time t and $\exists\, \hat{t}$ so that $U^i_{\hat{t}} > U^j_{\hat{t}}$

intertemporal preferences are binary relations R, P, and I that rank utility streams

- $U^i \; R \; U^j$ means U^i is at least as good as U^j
- $U^i \; P \; U^j$ means U^i is strictly preferred to U^j
- $U^i \; I \; U^j$ means indifferent between U^i and U^j

Discounted Utility: $V = \int_0^\infty U(t)e^{-\delta t}dt$



The red stream is preferred to the blue under discounted utility ($V^{\text{red}} > V^{\text{blue}}$)

The Popularity of Discounted Utility

$$V = \int_0^\infty U(t)e^{-\delta t}dt$$

Koopmans (1960): discounted utility with constant discount rate is the only criterion that satisfies the axioms of *continuity*, *sensitivity*, *stationarity* and *separability*, for the class of stationary ordinal utility functions.

- optimization problems under discounted utility are usually well behaved (e.g. V is finite)
- discount rate between any two periods is δ, only one parameter to calibrate
- optimal decisions are time consistent (i.e. they do not change if re-evaluated along the path)

Desirable Axioms of Intertemporal Preferences

1. *Completeness*

$$\text{for all } U^i \text{ and } U^j, \text{ either } U^i \; R \; U^j \text{ or } U^j \; R \; U^i$$

2. *Transitivity*

$$\text{for all } U^i, U^j, U^k, \text{ if } U^i \; R \; U^j \text{ and } U^j \; R \; U^k, \text{ then } U^i \; R \; U^k$$

3. *Strong Pareto Efficiency*

$$\text{for all } U^i, U^j, \text{ if } U^i \geq U^j, \text{ then } U^i \; R \; U^j, \text{ and if } U^i > U^j, \text{ then } U^i \; P \; U^j$$

Lauwers (1997): the discounted utility criterion satisfies completeness, transitivity, and strong Pareto efficiency.

Desirable Axioms of Intergenerational Preferences

Intergenerational equity requires that the position of a generation in time should not affect the way its utility is considered in the evaluation criterion.

Anonimity

Consider the utility stream

$$U \equiv (U_1, U_2, ..., U_{\tau-1}, U_\tau, U_{\tau+1}, ..., U_{t-1}, U_t, U_{t+1}, ...)$$

If a sequence of utility U' is obtained by a permutation of elements in U, e.g.

$$U \equiv (U_1, U_2, ..., U_{\tau-1}, \mathbf{U_t}, U_{\tau+1}, ..., U_{t-1}, \mathbf{U}_\tau, U_{t+1}, ...)$$

then $U \; I \; U'$, that is the evaluation must rank the two streams equivalently.

Impossibility

Diamond-Basu-Mitra There exists no numerically representable (complete) preference rule over infinite utility streams that satisfies both the *Strong Pareto* axiom (efficiency), and the *Anonimity* axiom (intergenerational equity).

ongoing research effort is in finding criteria that relax axioms but maintain elements of efficiency and intergenerational equity

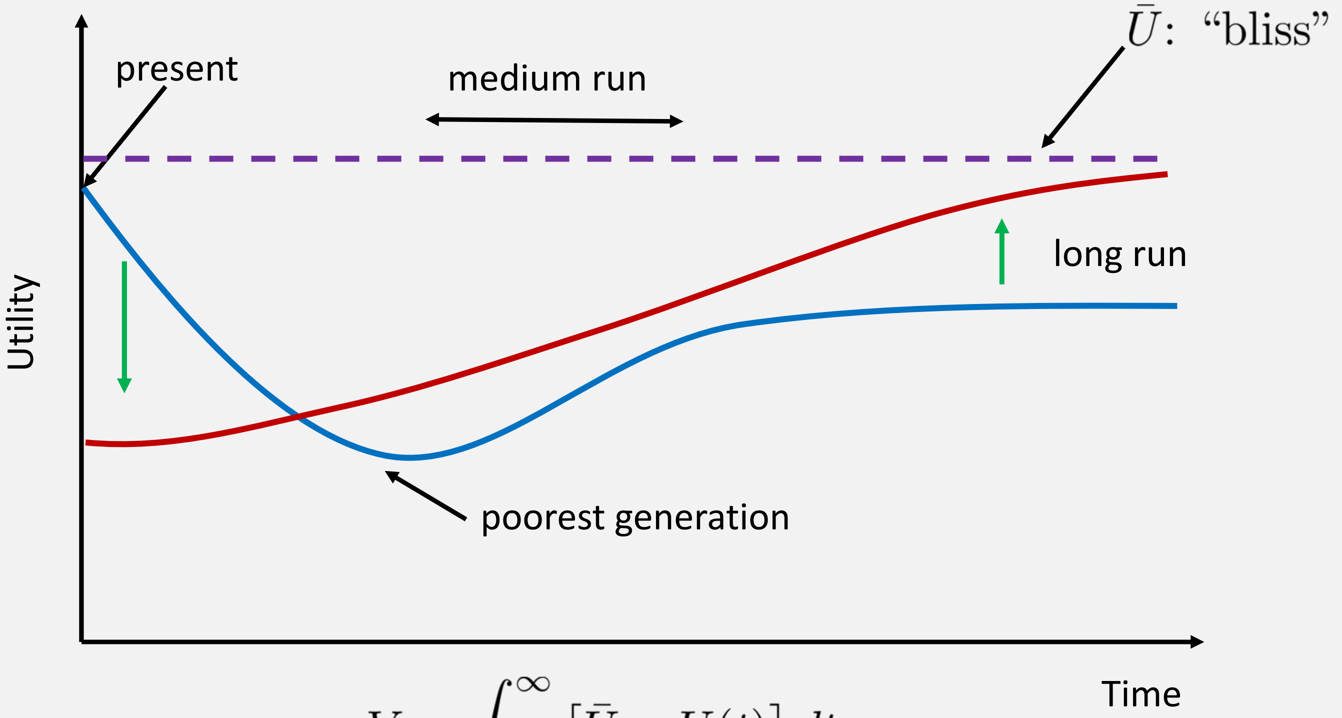
See Asheim (2010), Intergenerational Equity, *Annual Review*

Alternative Criteria

1. Undiscounted Utility - Ramsey criterion
2. Maximin - Rawls criterion
3. Green Golden Rule
4. Chichilnisky criterion

we first look at their definition and features, then we apply them to models

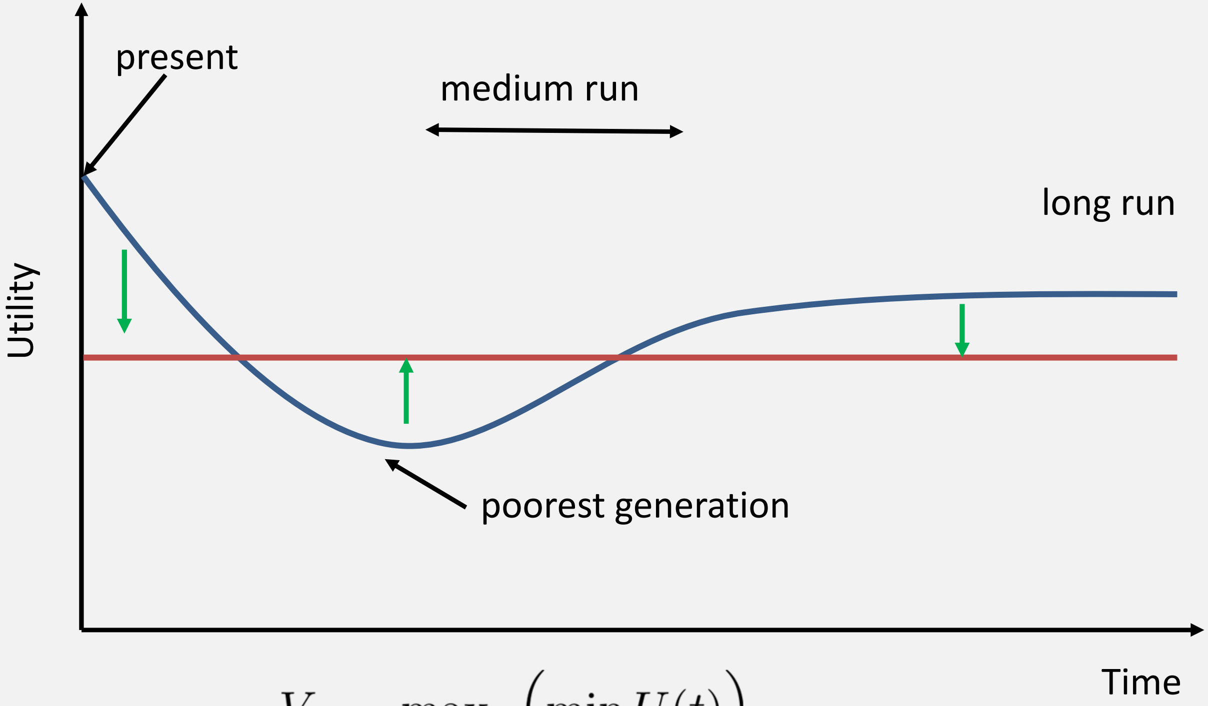
1. Undiscounted Utility – Ramsey Criterion



$$V = \int_0^\infty \left[\bar{U} - U(t)\right] dt$$

The red stream is preferred to the blue under Ramsey ($V^{\text{red}} > V^{\text{blue}}$)

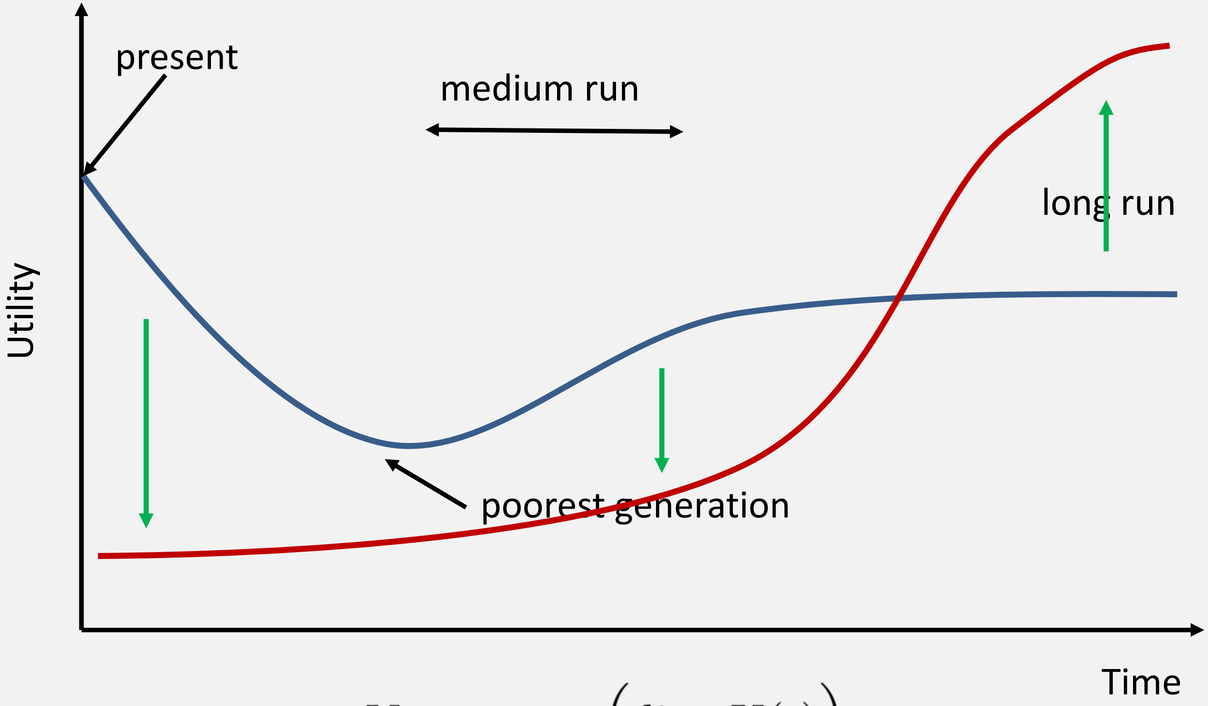
Maximin – Rawls Criterion



$$V = \max_{\text{controls}} \left(\min_t U(t) \right)$$

The red stream is preferred to the blue under Rawls ($V^{\text{red}} > V^{\text{blue}}$)

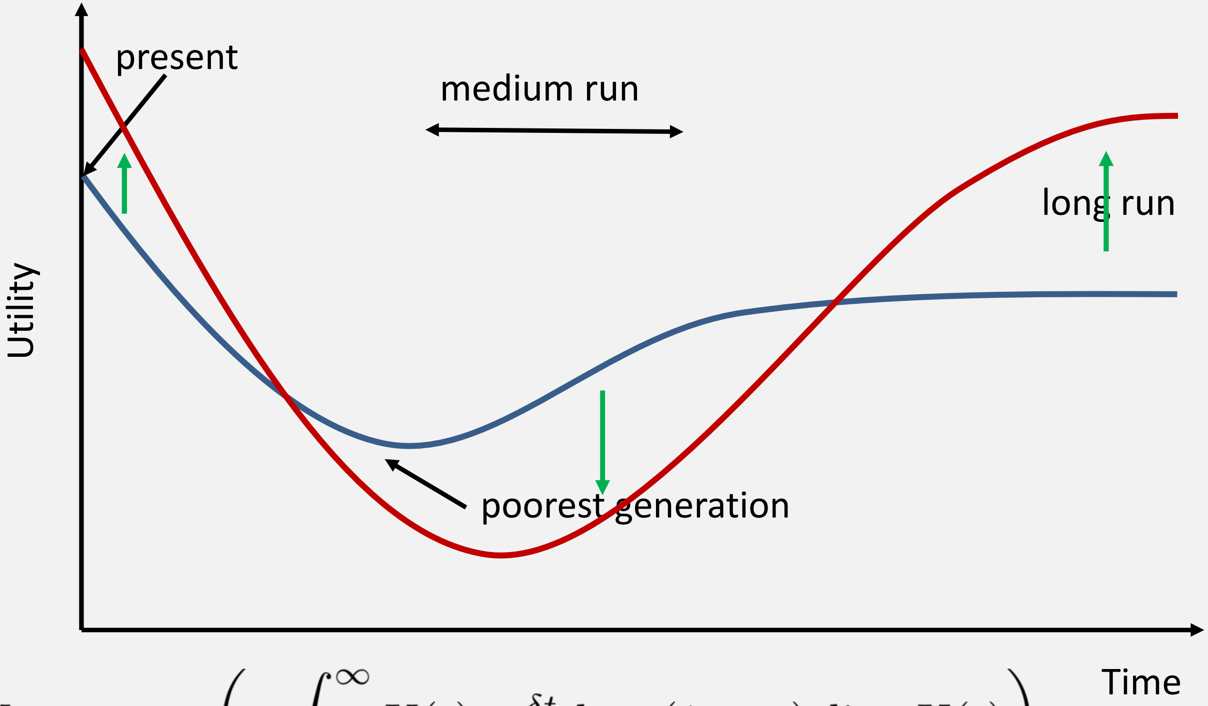
Green Golden Rule (GGR)



$$V = \max_{\text{controls}} \left(\lim_{t \to \infty} U(t) \right)$$

The red stream is preferred to the blue under GGR ($V^{\text{red}} > V^{\text{blue}}$)

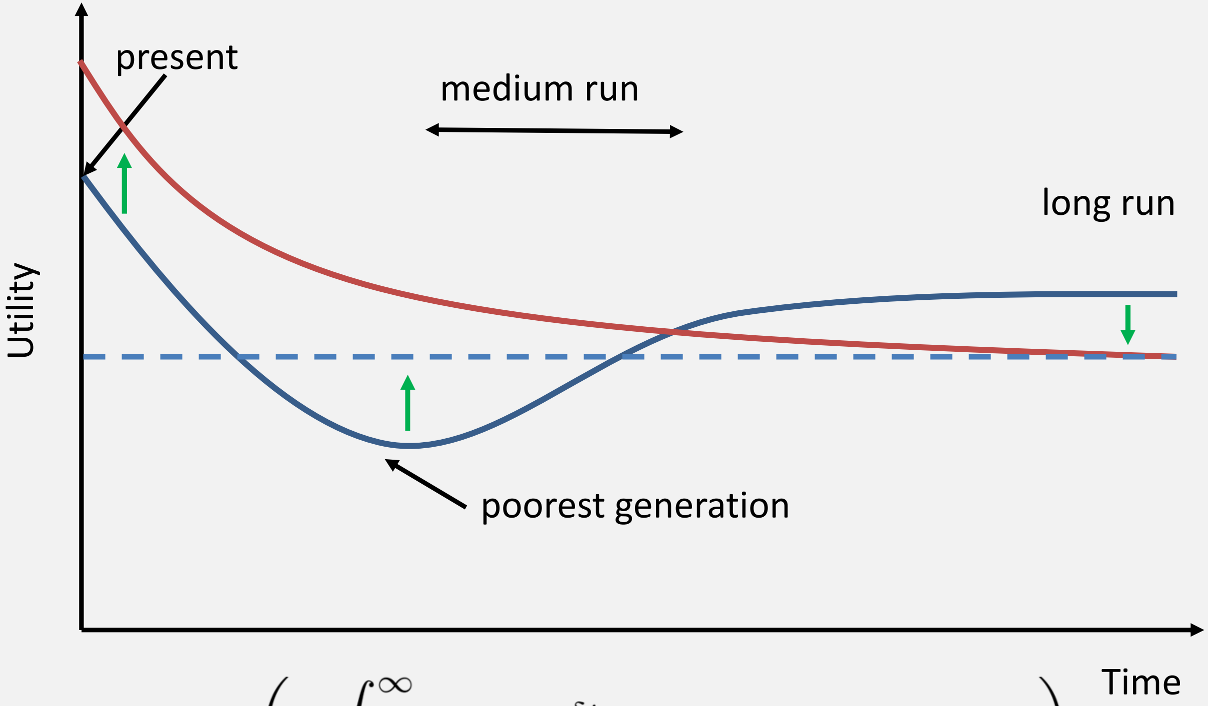
Chichilnisky Criterion (CC)



$$V = \max_{\text{controls}} \left(\omega \int_0^\infty U(t) e^{-\delta t} dt + (1-\omega) \lim_{t \to \infty} U(t) \right)$$

The red stream is preferred to the blue under CC ($V^{\text{red}} > V^{\text{blue}}$)

Bentham-Rawls Criterion (BR)



$$V = \max_{\text{controls}} \left(\omega \int_0^\infty U(t) e^{-\delta t} dt + (1-\omega) \min_t U(t) \right)$$

The red stream is preferred to the blue under BR ($V^{\text{red}} > V^{\text{blue}}$)

Issues with Alternative Criteria

- Ramsey: solution might not exist, needs finite bliss
- Rawls: dictatorship of poorest generation, needs “regularity” (technical)
- Green Golden Rule: dictatorship of the future
- Chichilnisky criterion: solution might not exist, poorest generation poorer

let us apply them to models of nonrenewable and renewable resources

Model 1: The "Cake-Eating" Economy

$$\max_{c(\cdot)} \int_0^\infty u(c(t))e^{-\delta t}dt$$

$$\dot{S}(t) = -c(t), \quad S(t) \geq 0 \text{ and } S(0) \text{ given}$$

The “Cake-Eating” Economy

$$\max_{c(\cdot)} \int_0^\infty u(c(t))e^{-\delta t}dt$$

$$\dot{S}(t) = -c(t), \quad S(t) \geq 0 \text{ and } S(0) \text{ given}$$

$$\mathcal{H}(c, S, q) = u(c(t)) + q(t)\dot{S}(t) = u(c(t)) - q(t)c(t)$$

$$c : \frac{\partial \mathcal{H}}{\partial c} = 0 \rightarrow u'(c(t)) - q(t) = 0$$

$$S : \frac{\partial \mathcal{H}}{\partial S} = \delta q(t) - \dot{q}(t) \rightarrow \delta q(t) - \dot{q}(t) = 0$$

$$q : \frac{\partial \mathcal{H}}{\partial q} = 0 \rightarrow c(t) = -\dot{S}(t)$$

$$TVC : \lim_{t \to \infty} e^{-\delta t} q(t)S(t) = 0$$

“transversality condition”

The "Cake-Eating" Economy

c) what is a key feature of co-state variable along the optimal path?

$$\delta q(t) - \dot{q}(t) = 0 \rightarrow \frac{\dot{q}(t)}{q(t)} = \delta$$

$$q(t) = q(0)e^{\delta t}$$

Hotelling Rule: along the efficient (optimal) extraction path, the resource shadow price increases at the discount rate.

The "Cake-Eating" Economy

d) characterized the optimal path for consumption $c(t)$

$$u'(c(t)) = q(t) \rightarrow \quad \frac{\dot{q}(t)}{q(t)} = \dot{c}(t)\frac{u''(c(t))}{u'(c(t))} \quad \rightarrow \quad \frac{\dot{q}(t)}{q(t)} = -\eta\frac{\dot{c}(t)}{c(t)}$$

hence:
$$\frac{\dot{c}(t)}{c(t)} = -\frac{\delta}{\eta}$$

full path: $c(t) = c(0)e^{-(\delta/\eta)t}$ where $c(0) = (\delta/\eta)S(0)$

The "Cake-Eating" Economy: Details

solve the ordinary differential equation

$$\dot{S}(t) = -c(0)e^{-(\delta/\eta)t} \rightarrow \quad S(t) = A + (\eta/\delta)c(0)e^{-(\delta/\eta)t}$$

$$S(0) = A + (\eta/\delta)c(0) \rightarrow A = S(0) - (\eta/\delta)c(0)$$

$$S(t) = S(0) - (\eta/\delta)c(0) + (\eta/\delta)c(0)e^{-(\delta/\eta)t}$$

use TVC to pick initial point of the path, $c(0)$:

$$TVC : \lim_{t\rightarrow\infty} e^{-\delta t}q(t)S(t) = 0$$

$$\lim_{t\rightarrow\infty} q(0)(S(0) - (\eta/\delta)c(0) + (\eta/\delta)c(0)e^{-(\delta/\eta)t}) = 0$$

$$(S(0) - (\eta/\delta)c(0)) = 0 \rightarrow \ c(0) = (\delta/\eta)S(0)$$

The "Cake-Eating" Economy under Alternative Criteria

1. Undiscounted Utility - Ramsey criterion: **no solution**

2. Maximin - Rawls criterion: **no solution**

3. Green Golden Rule: **all consumed by last generation that never comes**

4. Chichilnisky criterion: **no solution**

Valuing Environmental Assets

" Finding a framework that allows for a complete integration of the ways in which environmental assets contribute to the economy is a central part of the research agenda." Heal (1998, p. 14)

examples:

1. *biodiversity*: stock of knowledge, insurance role
2. *forests*: oxygen, climate stabilizer
3. *soil*: water purification, fertilization

environmental assets may have instrinsic value, independent of humanity

"A natural next step in environmental research is to study more closely the technologies and process by whith the stock of an environmental asset provides value to the community." Heal (1998, p. 19)

Modeling the Valuing of Environmental Assets

A stock of evironmental assets provides a flow of ecosystem services, but it is not only the size of the flow that matters for well-being: the size of the stock matters as well, in complex and yet to be fully understood ways. We capture this idea by a crude modeling choice:

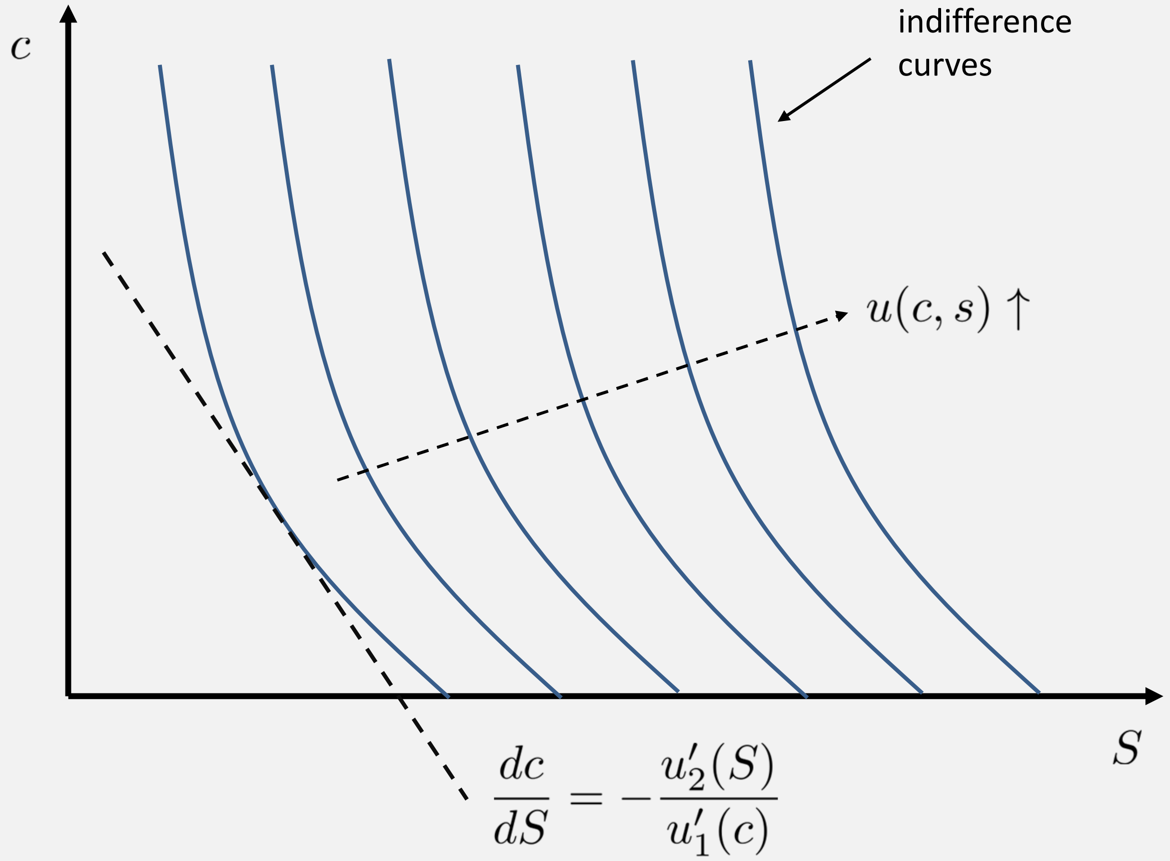
$$u(c, S)$$

Assumptions:

$$\lim_{c \to 0} \frac{\partial u(c, S)}{\partial c} < \infty \qquad\qquad u(0, s) > -\infty \quad \forall s$$

for simplicity of exposition: $u(c, s) = u_1(c) + u_2(s)$

Valuing the Depletable Stock



Model 2: Valuing the Cake in the Cake-Eating Economy

$$\max_{c(\cdot)} \int_0^\infty u(c(t), S(t)) e^{-\delta t} dt$$

$$\dot{S}(t) = -c(t), \quad S(t) \geq 0 \;\text{ and }\; S(0) \text{ given}$$

Model 2: Valuing the Cake in the Cake-Eating Economy

$$\mathcal{H} = u(c(t), S(t)) - q(t)c(t) + \phi(t)c(t)$$

$$u_1'(c(t)) = q(t) - \phi(t) \quad \begin{cases} u_1'(c(t)) \leq q(t) & \text{when} \quad c(t) = 0 \\ u_1'(c(t)) = q(t) & \text{when} \quad c(t) > 0 \end{cases}$$

$$\delta q(t) - \dot{q}(t) = u_2'(S(t)) \rightarrow \quad \frac{\dot{q}(t)}{q(t)} = \delta - \frac{u_2'(S(t))}{q(t)} \qquad \text{Hotelling Rule}$$

$$TVC : \lim_{t \to \infty} e^{-\delta t} q(t) S(t) = 0$$

Model 2: Valuing the Cake in the Cake-Eating Economy

Can the stock $S(t)$ be sustained along an optimal solution?

Let's study the stationary solution and see when $\lim_{t\to\infty} S(t) = \hat{S} > 0$.

$\dot{c}(t) = \dot{q}(t) = \dot{S}(t) = 0$

note: $\hat{c} = 0$ in the stationary solution (why?)

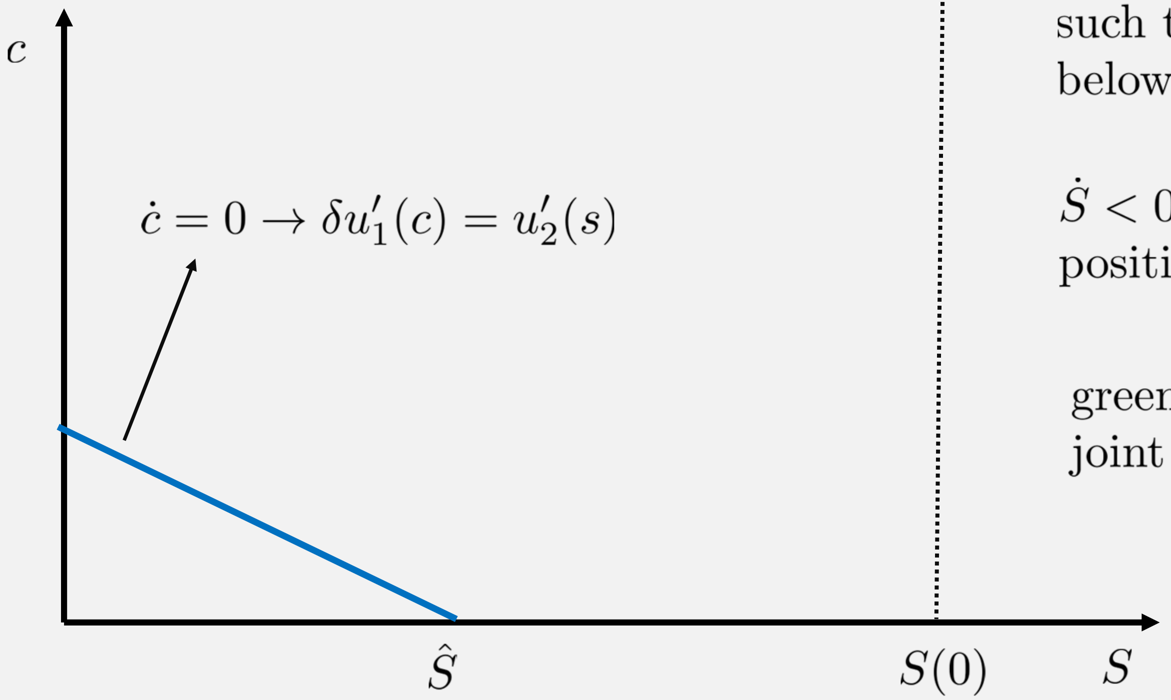
then:

$$u_1'(0) \leq \frac{u_2'(\hat{S})}{\delta}$$

interpretation: $u_1'(0)\Delta c \leq \dfrac{u_2'(\hat{S})}{\delta}\Delta c$

recall: $\frac{A}{\delta} = \int_0^\infty A e^{-\delta t} dt$

Phase Diagram

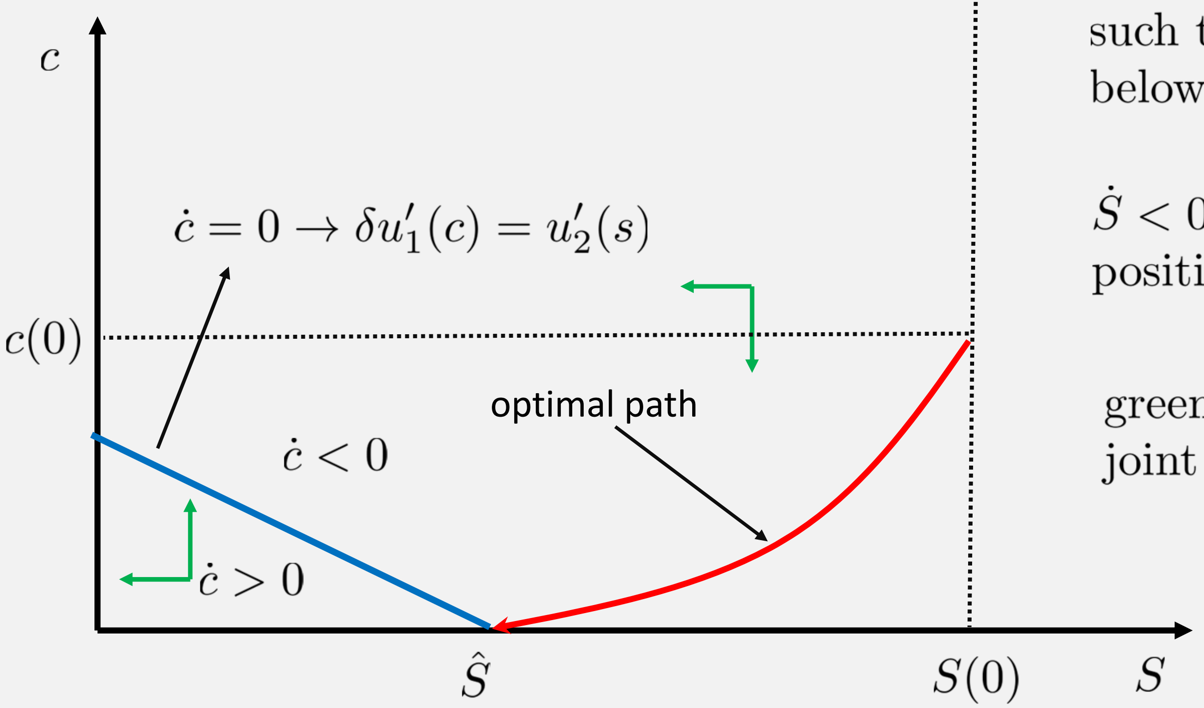


blue locus: combination of points such that $\dot{c} = 0$; above locus $\dot{c} < 0$, below locus $\dot{c} > 0$ (why?)

$\dot{S} < 0$ for all (c, S) in strictly positive quadrant (why?)

green arrows indicate joint dynamics of c and S

Phase Diagram



blue locus: combination of points such that $\dot{c} = 0$; above locus $\dot{c} < 0$, below locus $\dot{c} > 0$ (why?)

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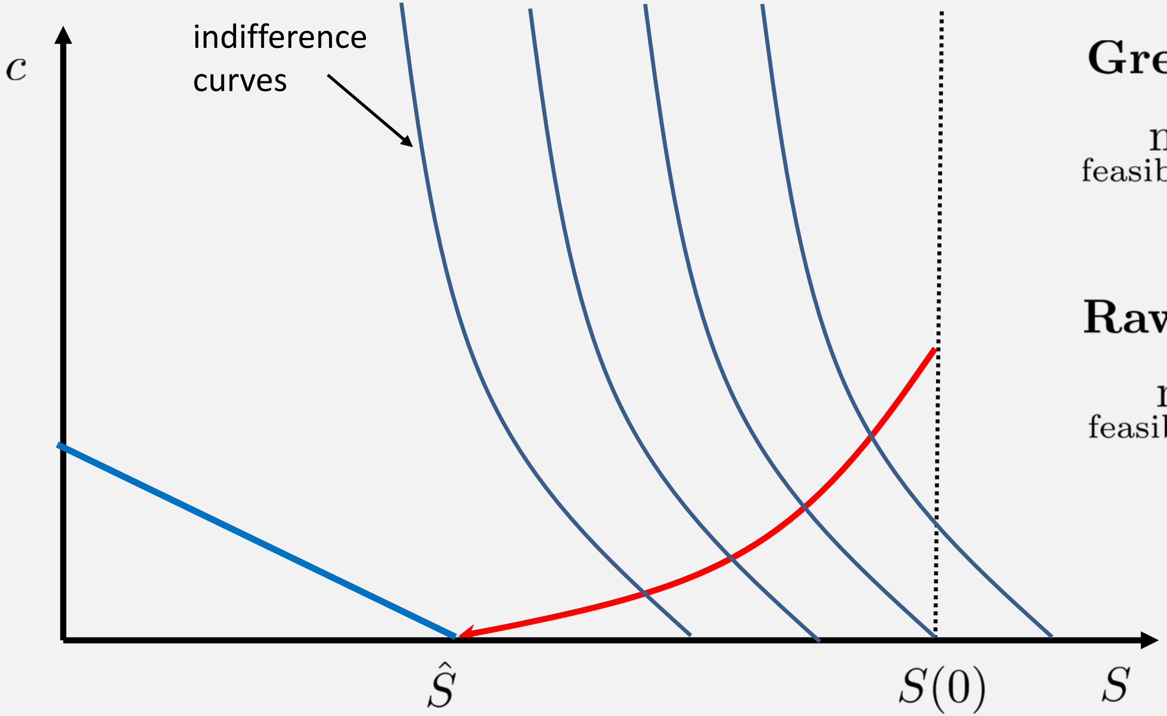
green arrows indicate joint dynamics of c and S

Solution under Alternative Criteria



Green Golden Rule

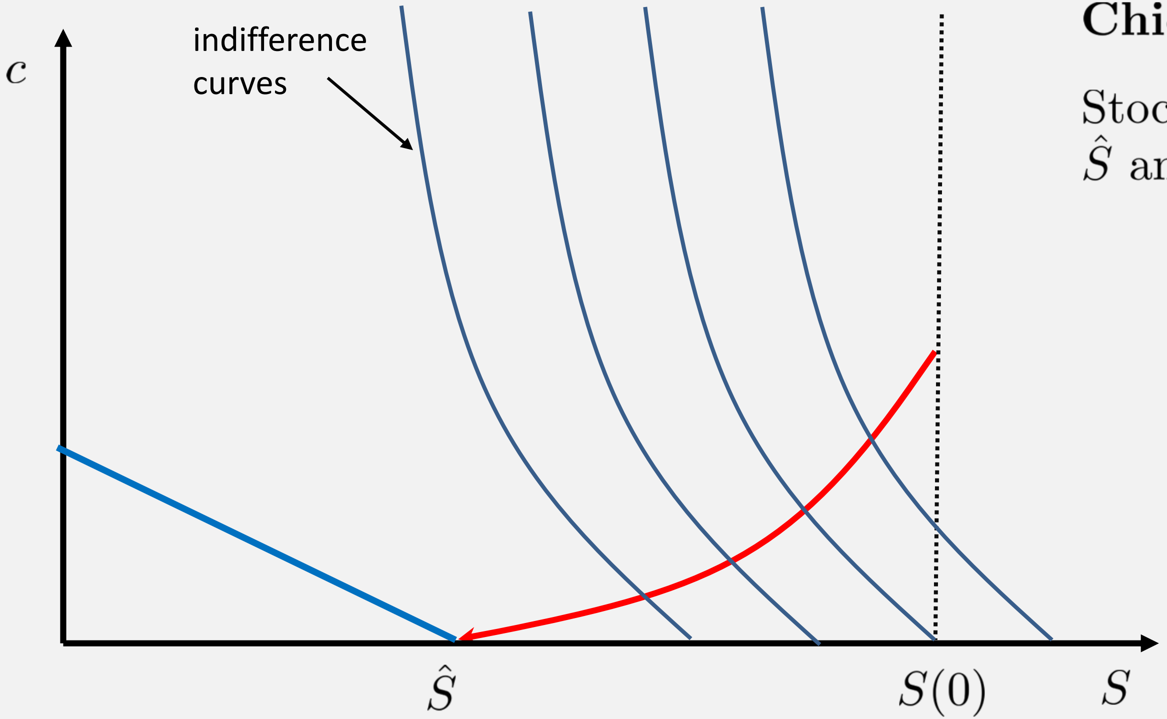
$$\max_{\text{feasible paths}} \left(\lim_{t \to \infty} u(c(t), S(t)) \right)$$

Rawlsian Criterion

$$\max_{\text{feasible paths}} \left(\min_{t} u(c(t), S(t)) \right)$$

Under both criteria the resource is fully preserved at $S(0)$!

Solution under Alternative Criteria



Chichilnisky Criterion

Stock preserved is inbetween $\hat{S}$ and $S(0)$, depending on ω

$$\max_{\text{feasible paths}} \left(\omega \int_0^\infty u(c(t), S(t)) e^{-\delta t} dt + (1-\omega) \min_t u(c(t), S(t)) \right)$$

Valuing the Cake (Model 2) under Alternative Criteria

1. Undiscounted Utility - Ramsey: **entire stock** $S(0)$ **preserved,** $c(t) = 0 \; \forall t$

2. Maximin - Rawls criterion: **entire stock** $S(0)$ **preserved,** $c(t) = 0 \; \forall t$

3. Green Golden Rule: **entire stock** $S(0)$ **preserved,** $c(t) = 0 \; \forall t$

4. Chichilnisky criterion: **stock preserved is between** $\hat{S}$ **and** $S(0)$

Renewable Resources

Introducing the possibility of resources being renewable is a further step towards integrating the role of environmental assets.

$$\dot{S}(t) = r(S(t)) - c(t)$$

$r(S(t))$ captures the growth of resources without human intervention; dynamics of econological systems can be introduced via a functional specification for $r(.)$.

Assumptions: $r(0) = 0$, there exists $\bar{S}$ such that $r(\bar{S}) = 0$, $r(.)$ is strictly concave and twice continuously differentiable.

example: in population biology, $r(.)$ is usually assumed to be quadratic, so growth of unexploited population is logistic.

Model 3: Renewable Resources

$$\max_{c(\cdot)} \int_0^\infty u(c(t), S(t)) e^{-\delta t} dt$$

$$\dot{S}(t) = r(S(t)) - c(t), \quad S(t) \geq 0 \text{ and } S(0) \text{ given}$$

optimal path must satisfy

$$u_1'(c(t)) = q(t)$$

$$\delta q(t) - \dot{q}(t) = u_2'(S(t)) + q(t) r'(S(t)) \rightarrow \frac{\dot{q}(t)}{q(t)} = \delta - r'(S(t)) - \frac{u_2'(S(t))}{q(t)}$$

Hotelling Rule

$$TVC: \lim_{t \to \infty} e^{-\delta t} q(t) S(t) = 0$$

Model 3: Renewable Resources

What determines the stock $S(t)$ that is sustained along an optimal solution?

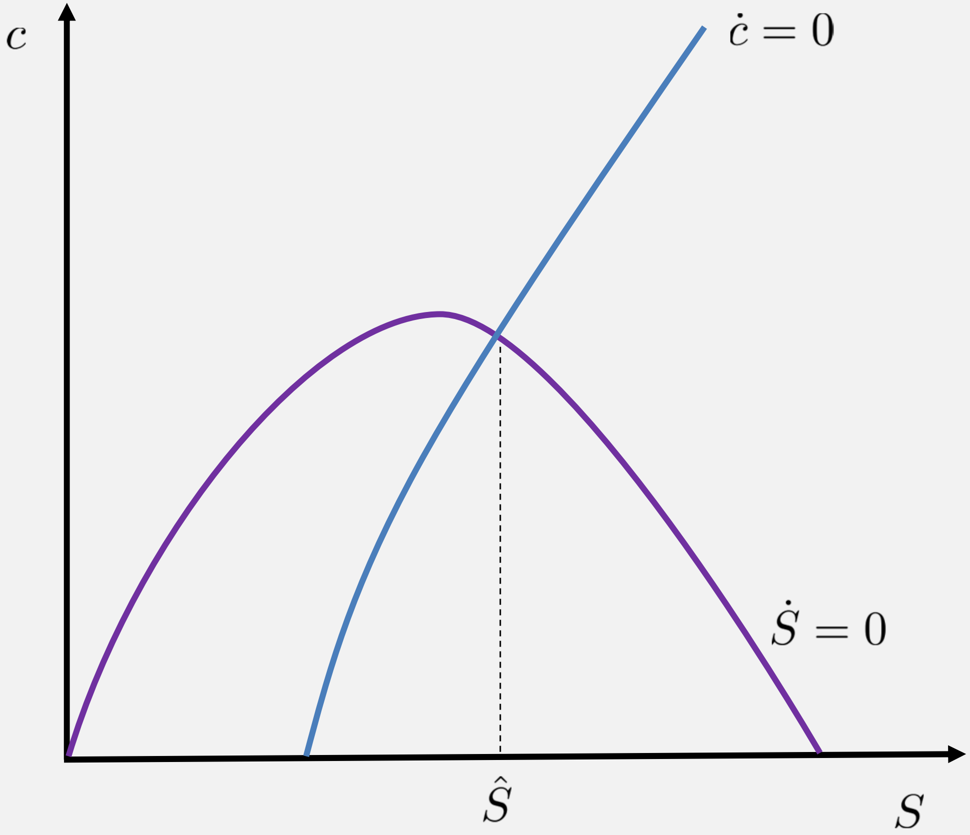
Stationary Solution: $\dot{c}(t) = \dot{q}(t) = \dot{S}(t) = 0$

$\dot{S} = 0 \rightarrow \hat{c} = r(\hat{S})$, hence $\hat{c} > 0$ is possible

$$\dot{q} = 0 \rightarrow \quad \frac{u_2'(\hat{S})}{u_1'(\hat{c})} = \delta - r'(\hat{S})$$

interpretation: $$u_1'(\hat{c})\Delta c = \frac{u_2'(\hat{S}) + r'(\hat{S})u_1'(\hat{c})}{\delta}\Delta c$$

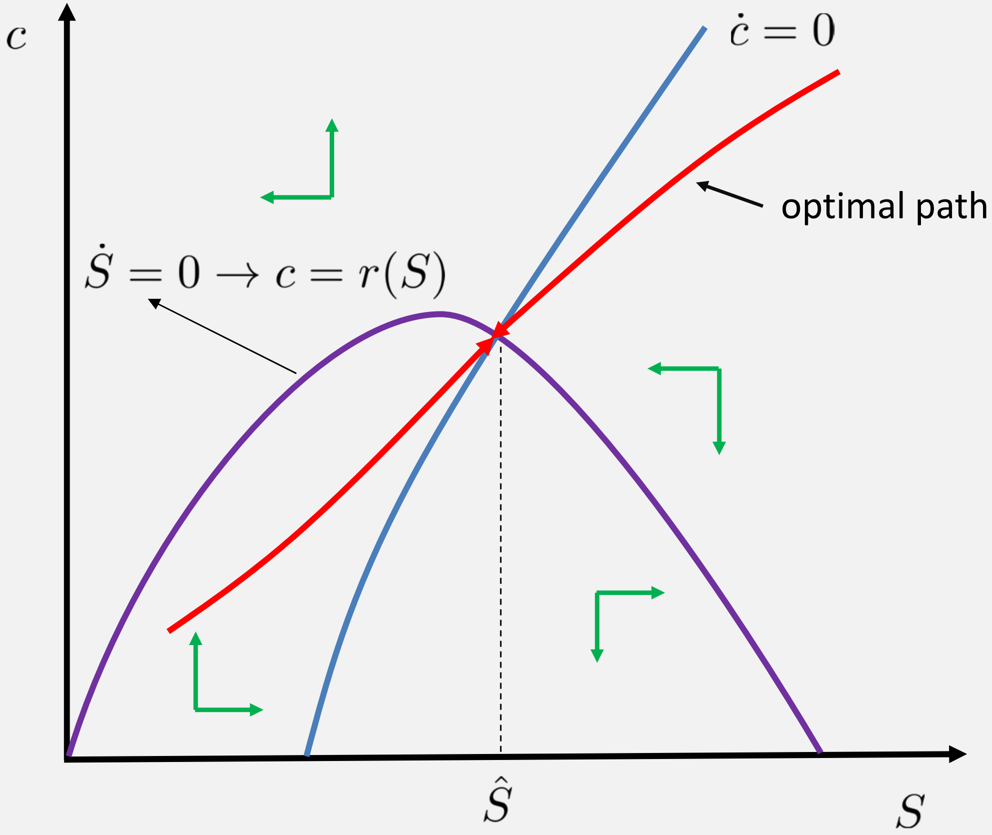
Phase Diagram



purple locus: combination of points such that $\dot{S} = 0$; above locus $\dot{S} < 0$, below locus $\dot{S} > 0$ (why?)

blue locus: combination of points such that $\dot{c} = 0$; above locus $\dot{c} > 0$, below locus $\dot{c} < 0$ (why?)

Phase Diagram



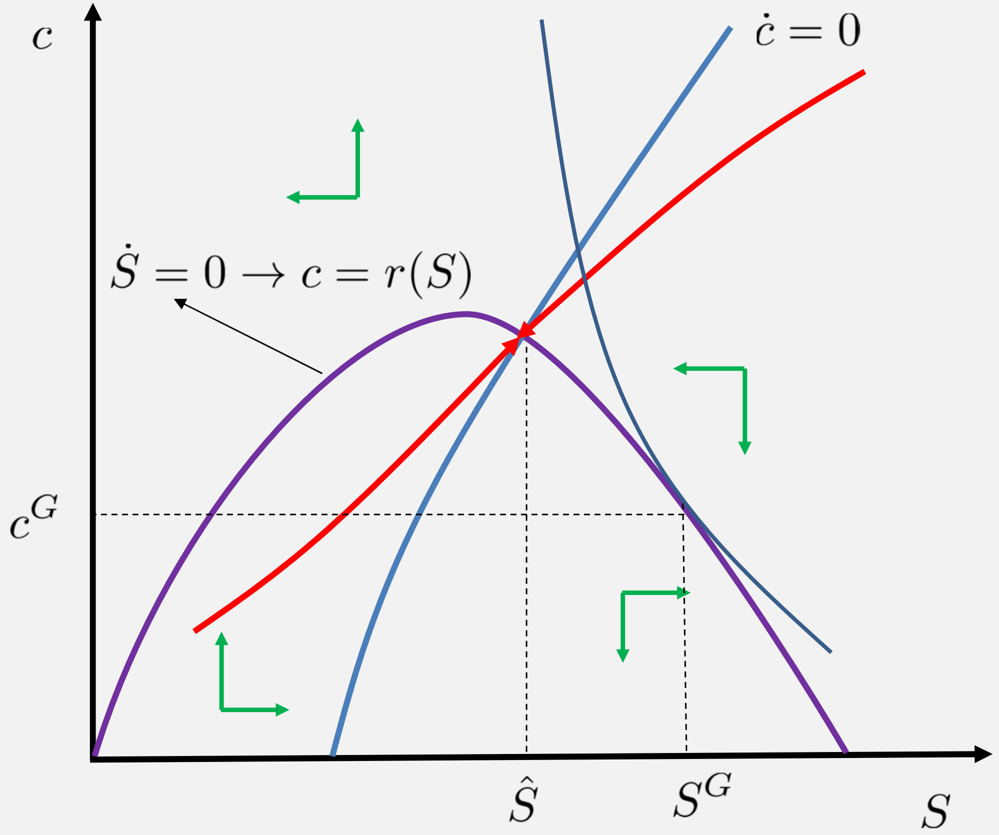
important features of solution:

- both consumption and resource stock can be increasing along utilitarian solution
- depending on $S(0)$ optimal to deplete or accumulate, towards $\hat{S}$

policy-relevant:

1. how do we know what is $\hat{S}$?
2. are resources priced so that utilization rate is socially optimal?

Solution Under the Green Golden Rule



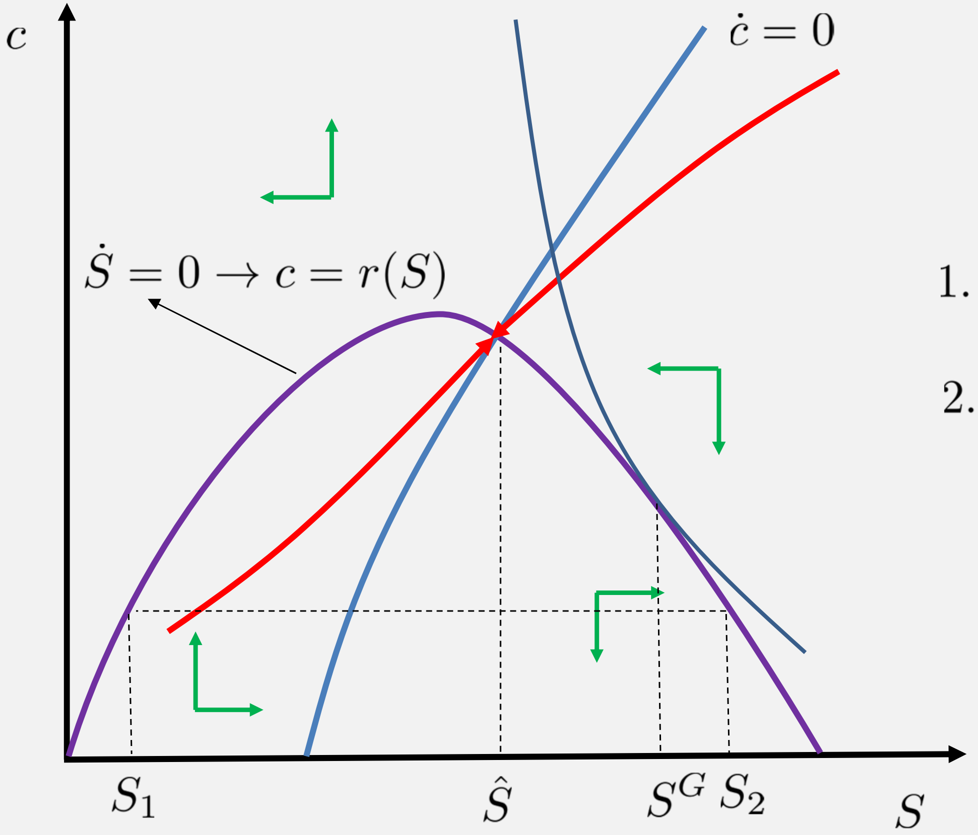
Green Golden Rule

$$\max_{\text{feasible paths}} \left(\lim_{t \to \infty} u(c(t), S(t)) \right)$$

$$\begin{cases} \dfrac{u_2'(S^G)}{u_1'(c^G)} = -r'(S^G) \\ c^G = r(S^G) \end{cases}$$

problem: path to S^G not determined!

Solution Under the Rawls Criterion



Rawls Criterion

$$\max_{\text{feasible paths}} \left(\min_t u(c(t), S(t)) \right)$$

1. if $S(0) < S^G$, $c^R = r(S(0))$
2. if $S(0) \geq S^G$, $c^R = r(S^G)$

Chichilnisky Criterion

A solution under the Chichilnisky criterion in the renewable resources problem with a constant discount rate, $e^{-\delta t}$, **does not exist**.

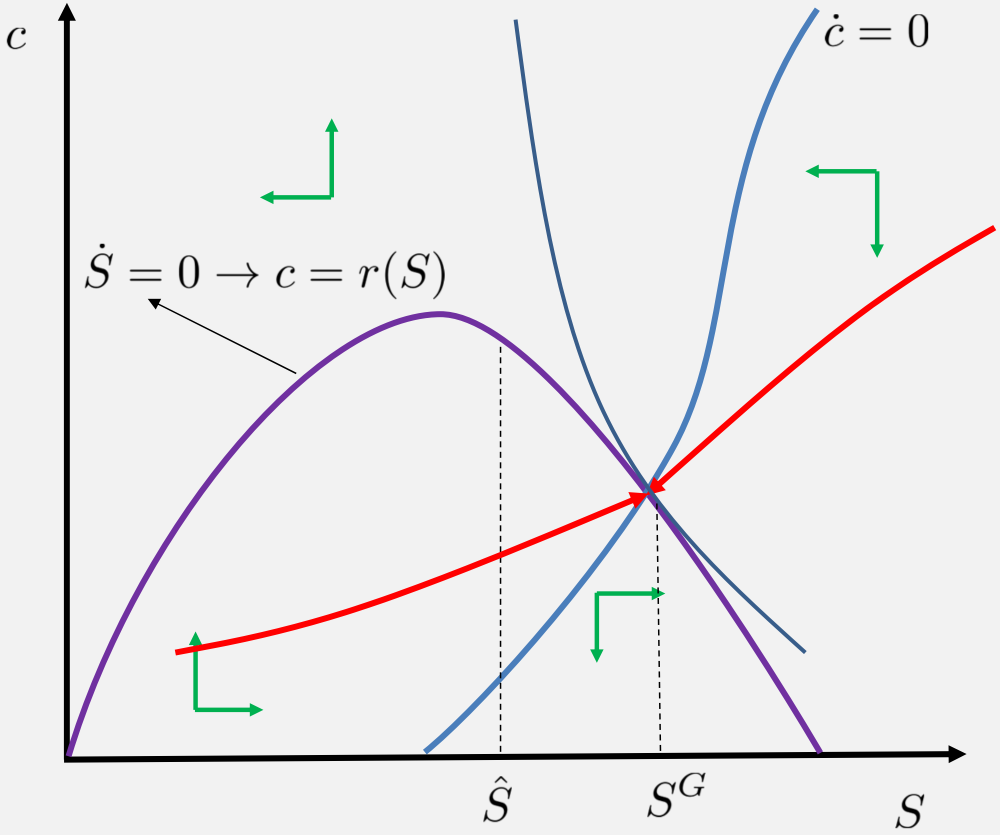
If the discounting is modified so that the discount rate is not constant, but it declines to zero, then a solution exists and it asymptotes towards the Green Golden Rule.

Example: $\delta(t) = -\frac{\alpha}{t}$, so discount factor is, $e^{-\alpha \ln t} = t^{-\alpha}$, with $\alpha > 1$

Weber-Fechner law: human response to a change in a stimulus is inversely proportional to the preexisting stimulus.

The response to a change in futurity (say postponement of a year) is inversely proportional to the distance into the future when the change happens. This corresponds to a decreasing discount rate.

Solution with Chichilnisky Criterion and Declining Discount



under a declining discount rate the Chichilnisky criterion has a solution with a path that converges to the stock under the Green Golden Rule

note: path to S^G is determined!

Renewable Resources (Model 3) under Alternative Criteria

1. Undiscounted Utility - Ramsey: $\bar{U} = u(r(S^R), S^R)$, $S^R \lesseqgtr S(0)$

2. Maximin - Rawls criterion: $S(0)$, **if** $S(0) < S^G$, **or** S^G **preserved**

3. Green Golden Rule: **positive stock** S^G **preserved, undetermined path**

4. Chichilnisky criterion: **no solution, unless declining** $\delta(t)$ **assumed**

Taking Stock and Next Steps

- no dominating criterion to evaluate stream of well-being of generations that ensures both *efficiency* and *intergeneretional equity.*

- some criteria lead to conservation of environmental assets, but path towards long-run conservation is left undetermined.

- the question of what exactly has to be preserved and sustained, and how, does not have an unequivocal answer.

next step: model humand-made capital and natural resources

Homework: the Dasgupta-Heal Model

$$\max_{c(\cdot),r(\cdot)} \int_0^\infty u(c(t))e^{-\delta t}dt$$

$$\dot{K}(t) = F(K(t), r(t)) - c(t),$$

$$\dot{S}(t) = -r(t), \quad K(0) \text{ and } S(0) \text{ given}$$

$$F(K, r) = K^\alpha r^\beta$$

a) write the Hamiltonian associated with the maximization problem

b) obtain the necessary conditions for an optimal solution

c) can a positive consumption be maintained in the very long run?

d) what is the Hotelling rule in this economy?