IAERE Summer School 2022
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The Macroeconomic Theory of Sustainability: An Introduction

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Lecture 3

Overview of Lectures

Lecture 1:

i. review of dynamic decision-making

ii. efficiency and intergenerational equity

iii. macro approach to sustainability

Lecture 2:

i. sustainability criteria

ii. conservation of non-renewable and renewable resources

Lecture 3:

i. natural resources and reproducible capital

ii. sustainability and national accounting

Summary of Lecture 2

- sustainable well-being requires intergenerational equity
- impossibility result: there are no complete intergenerational preferences representation that ensure both efficiency and intergenerational equity
- "alternative" criteria to discounted utilitarianism

  undiscounted utility - Ramsey

  maximin - Rawls

  Green Golden Rule

  Chichilnisky

- applied to model of nonrenewable resources where environmental assets are directly valued
- result: environmental stock fully preserved at the expense of consumption

Model 3: Renewable Resources

$$\dot{S}(t) = r(S(t)) - c(t),$$

$r(S(t))$: embeds ecological/biological/climate dynamics

accommodates non-convex ecosystems, flips, hysteresis, irreversibility, etc.

see Dasgupta and Maler (2003), *The Economics of Non-Convex Ecosystems*

Model 3: Renewable Resources

$$\max_{c(\cdot)} \int_0^\infty u(c(t), S(t)) e^{-\delta t} dt$$

$$\dot{S}(t) = r(S(t)) - c(t), \quad S(t) \geq 0 \text{ and } S(0) \text{ given}$$

optimal path must satisfy

$$u_1'(c(t)) = q(t)$$

$$\delta q(t) - \dot{q}(t) = u_2'(S(t)) + q(t) r'(S(t)) \rightarrow \frac{\dot{q}(t)}{q(t)} = \delta - r'(S(t)) - \frac{u_2'(S(t))}{q(t)}$$

Hotelling Rule

$$TVC: \lim_{t \to \infty} e^{-\delta t} q(t) S(t) = 0$$

Model 3: Renewable Resources

What determines the stock $S(t)$ that is sustained along an optimal solution?

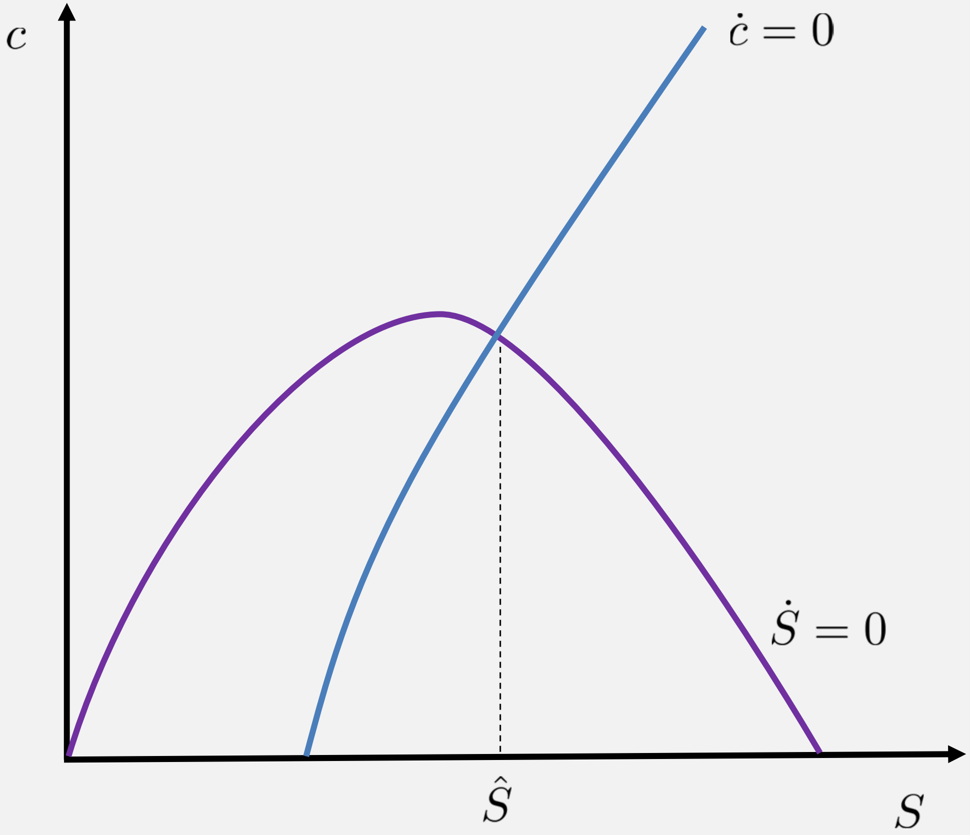
Stationary Solution: $\dot{c}(t) = \dot{q}(t) = \dot{S}(t) = 0$

$\dot{S} = 0 \rightarrow \hat{c} = r(\hat{S})$, hence $\hat{c} > 0$ is possible

$$\dot{q} = 0 \rightarrow \quad \frac{u_2'(\hat{S})}{u_1'(\hat{c})} = \delta - r'(\hat{S})$$

interpretation: $$u_1'(\hat{c})\Delta c = \frac{u_2'(\hat{S}) + r'(\hat{S})u_1'(\hat{c})}{\delta}\Delta c$$

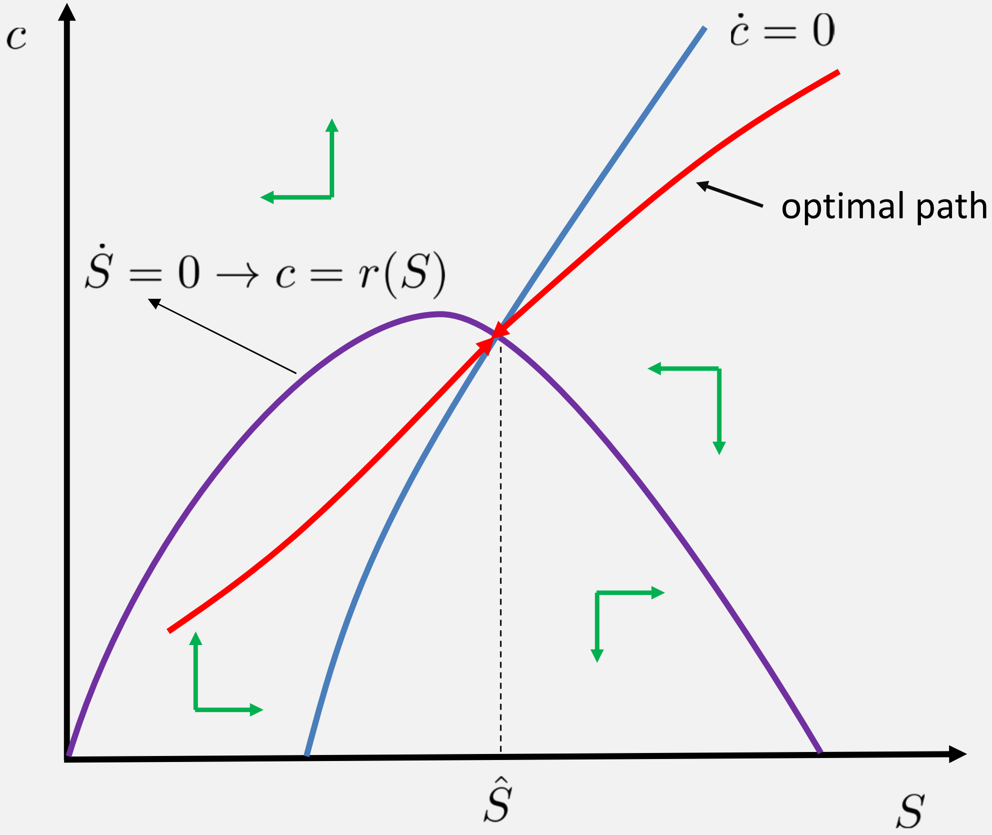
Phase Diagram



purple locus: combination of points such that $\dot{S} = 0$; above locus $\dot{S} < 0$, below locus $\dot{S} > 0$ (why?)

blue locus: combination of points such that $\dot{c} = 0$; above locus $\dot{c} > 0$, below locus $\dot{c} < 0$ (why?)

Model 3 Dynamics



important features of solution:

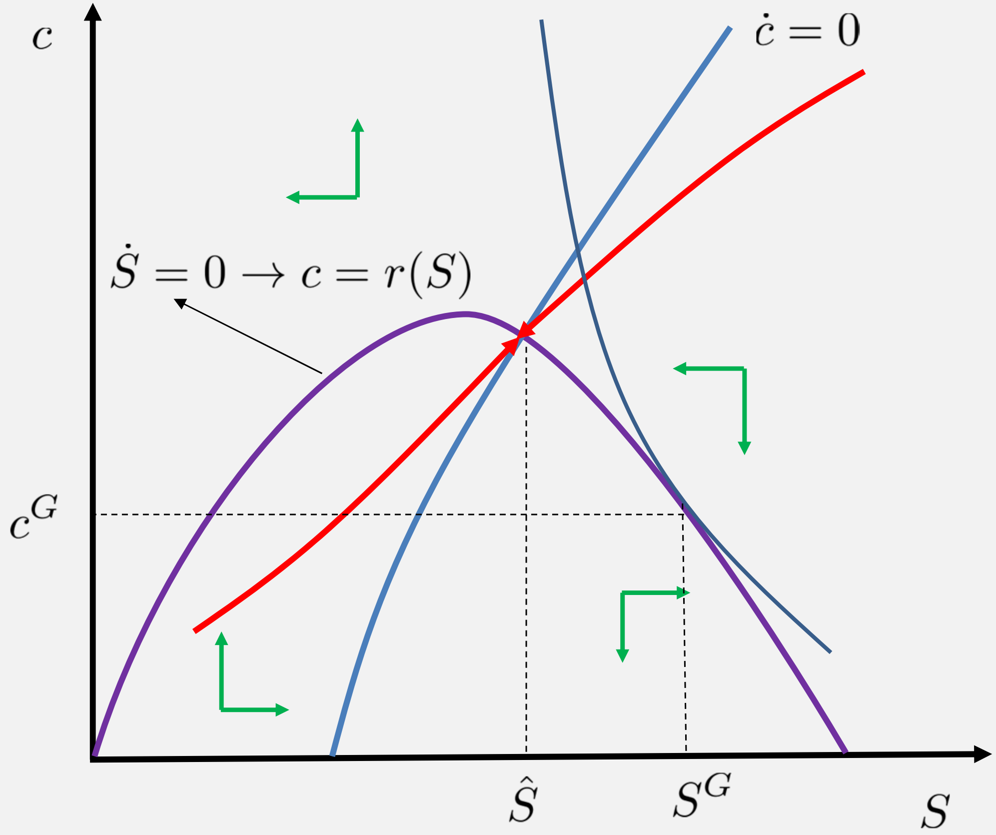
- both consumption and resource stock can be increasing along the utilitarian solution
- depending on $S(0)$ optimal to deplete or accumulate, towards $\hat{S}$

policy-relevant:

1. how do we know what is $\hat{S}$?
2. are resources priced so that utilization rate is socially optimal?

$$\frac{\dot{q}(t)}{q(t)} = \delta - r'(S(t)) - \frac{u_2'(S(t))}{q(t)}$$

Solution Under the Green Golden Rule



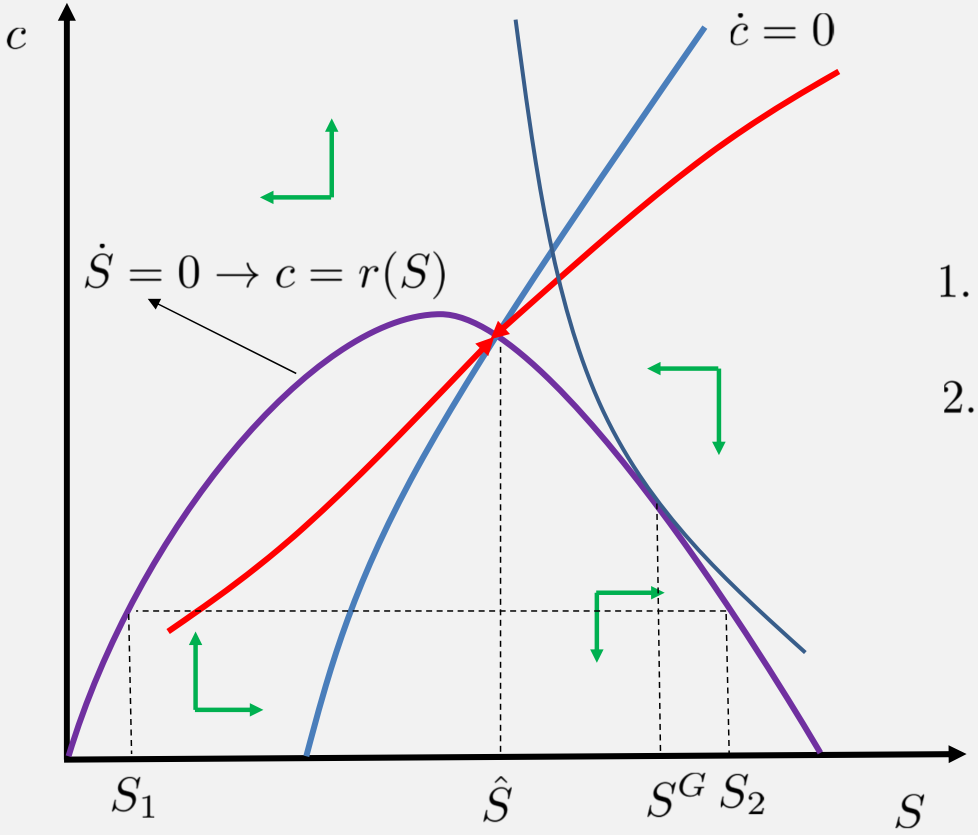
Green Golden Rule

$$\max_{\text{feasible paths}} \left(\lim_{t \to \infty} u(c(t), S(t)) \right)$$

$$\begin{cases} \dfrac{u_2'(S^G)}{u_1'(c^G)} = -r'(S^G) \\ c^G = r(S^G) \end{cases}$$

problem: path to S^G not determined!

Solution Under the Rawls Criterion



Rawls Criterion

$$\max_{\text{feasible paths}} \left(\min_t u(c(t), S(t)) \right)$$

1. if $S(0) < S^G$, $c^R = r(S(0))$

2. if $S(0) \geq S^G$, $c^R = r(S^G)$

Chichilnisky Criterion

$$V = \max_{\text{controls}} \left(\omega \int_0^\infty U(t) e^{-\delta t} dt + (1-\omega) \lim_{t \to \infty} U(t) \right)$$

A solution under the Chichilnisky criterion in the renewable resources problem with a constant discount rate, $e^{-\delta t}$, **does not exist**.

The criterion makes it optimal to stay on the utilitarian path as long as possible and then "jump" to the Green Golden Rule solution.

However, it always pays to stay an extra instant on the utilitarian path before jumping to the GGR. Hence, a solution does not exist.

Decreasing Discount Rate

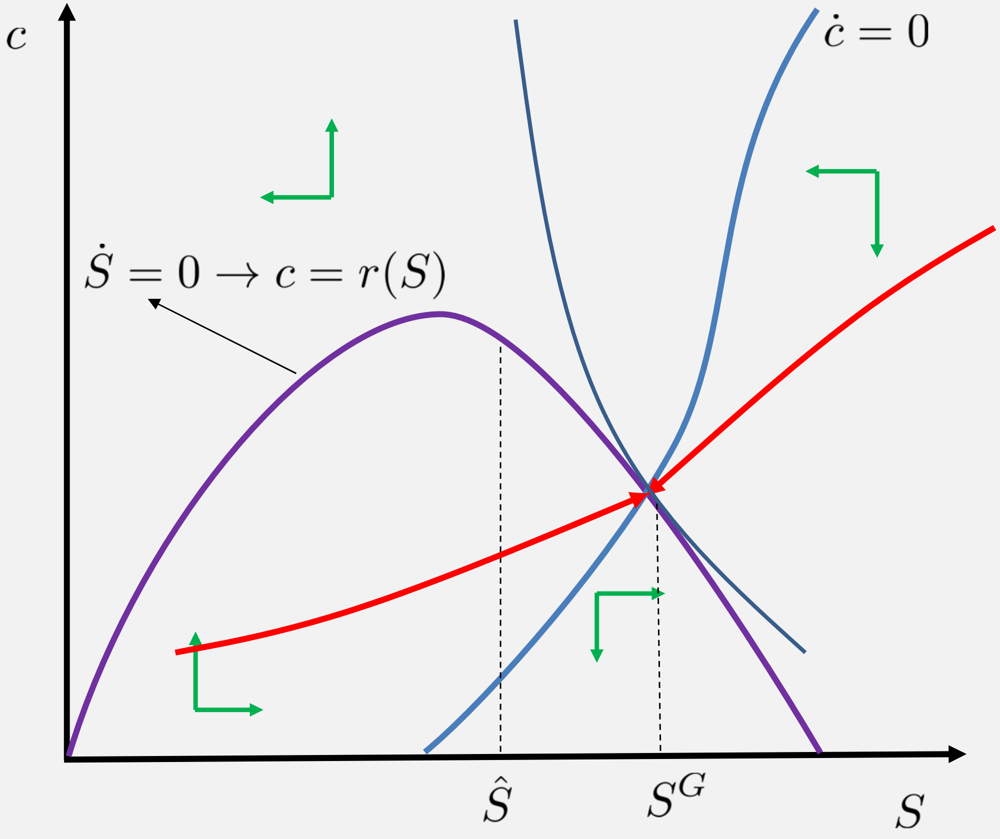
Weber-Fechner law: human response to a change in a stimulus is inversely proportional to the preexisting stimulus.

Implication for Discounting: The response to a change in futurity (say postponement of a year) is inversely proportional to the distance into the future when the change happens. This corresponds to a decreasing discount rate.

Example: $\delta(t) = -\frac{\alpha}{t}$, so discount factor is, $e^{-\alpha \ln t} = t^{-\alpha}$, with $\alpha > 1$

Result: If the discounting is modified so that the discount rate is not constant, but it declines to zero, then a solution under the Chichilnisky criterion exists and it asymptotes towards the Green Golden Rule.

Solution with Chichilnisky Criterion and Declining Discount



under a declining discount rate the Chichilnisky criterion has a solution with a path that converges to the stock under the Green Golden Rule

note: path to S^G is determined!

Renewable Resources (Model 3) under Alternative Criteria

1. Undiscounted Utility - Ramsey: $\bar{U} = u(r(S^R), S^R)$, **if it exists.**

2. Maximin - Rawls criterion: $S(0)$, **if** $S(0) < S^G$, **or** S^G **preserved**

3. Green Golden Rule: **positive stock** S^G **preserved, undetermined path**

4. Chichilnisky criterion: **no solution, unless declining** $\delta(t)$ **assumed**

Taking Stock and Next Steps

- no dominating criterion to evaluate stream of well-being of generations that ensures both *efficiency* and *intergeneretional equity.*

- some criteria lead to conservation of environmental assets, but path towards long-run conservation is left undetermined.

- the question of what exactly has to be preserved and sustained, and how, does not have an unequivocal answer.

next step: model humand-made capital and natural resources

Dasgupta-Heal-Stiglitz-Solow Model (DHSS)

$$\dot{K}(t) = F(K(t), r(t)) - c(t),$$

$$\dot{S}(t) = -r(t)$$

Properties of production function are important here:

$$F(K, r) = A\left[\alpha K^{\frac{\nu-1}{\nu}} + \beta r^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$$

1. $\nu > 1$: K and r are sustitutable, r is not essential, $F(K, 0) > 0$
2. $\nu < 1$: K and r are not-sustitutable, r is essential, $F(K, 0) = 0$
3. $\nu = 1$: K and r are sustitutable, r is essential, $F(K, 0) = 0$

Dasgupta-Heal

$$\max_{c(\cdot),r(\cdot)} \int_0^\infty u(c(t))e^{-\delta t}dt$$

$$\dot{K}(t) = F(K(t), r(t)) - c(t),$$

$$\dot{S}(t) = -r(t), \quad K(0) \text{ and } S(0) \text{ given}$$

$$F(K, r) = K^\alpha r^\beta$$

Dasgupta-Heal

$$\mathcal{H} = u(c(t)) + p(t)\left(F(K(t), r(t)) - c(t)\right) - q(t)r(t)$$

$$c : u'(c(t)) = p(t) \qquad r : p(t)F_r = q(t) \qquad F_r \equiv \partial F/\partial r$$

$$K : \delta p(t) - \dot{p}(t) = p(t)F_K \qquad S : \delta q(t) - \dot{q}(t) = 0 \qquad F_K \equiv \partial F/\partial K$$

$$\frac{\dot{p(t)}}{p(t)} = -\eta\frac{\dot{c(t)}}{c(t)}, \quad \text{and} \quad \frac{\dot{p(t)}}{p(t)} = \delta - F_K, \quad \text{so} \quad \frac{\dot{c(t)}}{c(t)} = \frac{F_K - \delta}{\eta}$$

result: as K accumulates, $F_K \downarrow$, eventually $\dot{c} < 0$, and $c(t) \to 0$

note: aversion to ingergenerational inequality η has usual impact on $\dot{c}$

Hotelling Rule in Production Economies

$$S : \delta q(t) - \dot{q}(t) = 0 \rightarrow \frac{\dot{q}(t)}{q(t)} = \delta \rightarrow q(t) = q(0)e^{-\delta t} \quad \text{price in terms of utils}$$

what about the price in terms of output (consumption)?

$$r : p(t)F_r = q(t) \rightarrow \frac{\dot{p}(t)}{p(t)} + \frac{\dot{F}_r}{F_r} = \frac{\dot{q}(t)}{q(t)}, \quad \text{recall} \quad \frac{\dot{p(t)}}{p(t)} = \delta - F_K,$$

hence

$$\frac{\dot{F}_r}{F_r} = F_K$$

Hotelling Rule: the resource price in terms of output (the marginal productivity of the resource F_r) must increase at a rate that is equal to the real interest rate (the marginal product of capital F_K).

Undiscounted Utility - Ramsey

$$\max_{c(\cdot),r(\cdot)} \int_0^\infty -c(t)^{1-\eta} dt \qquad \textbf{Ramsey criterion } (\bar{U} = 0)$$

$$\dot{K}(t) = K(t)^\alpha r(t)^\beta - c(t),$$

$$\dot{S}(t) = -r(t), \quad K(0) \text{ and } S(0) \text{ given}$$

result: solution exists if $\eta > (1-\beta)/(\alpha-\beta) > 1$

along the solution consumption, and thus utility, are *increasing*

note: unsustainability ($c(t) \to 0$) in production with nonrenewable resources is due to discounting and not to the resource being nonrenewable!

Dasgupta-Heal-Stiglitz: the Role of Technology

$$F(K(t), r(t)) = A(t)K(t)^{\alpha} r(t)^{\beta} L(t)^{1-\alpha-\beta}$$

$$A(t) = A(0)e^{\gamma t}$$

$$L(t) = L(0)e^{nt}$$

result: the solution is a balanced growth path with constant per-capita consumption when

$$\frac{\gamma}{n} > \beta$$

Green Golden Rule

solution depends on the role of resources in the production function

1. resource is essential: $F(K, 0) = 0$

   GGR: it is not possible to sustain a positive consumption in the long run, so the next best thing is to fully preserve the environmental asset at $S(0)$

2. resource is non-essential: $F(K, 0) > 0$

   GGR: capital accumulates to the golden rule of capital, and the environmental stock is fully preserved at $S(0)$.

3. if $F(K, r) = K^\alpha r^\beta$, GGR has no solution!

Chichilnisky Criterion

Leads to conservation of the stock of resources that is higher than the discounted utility criterion, but lower than the GGR (full preservation).

Dasgupta-Heal-Solow

Solow (1974): what is the highest level of constant consumption that can be sustained at all time in the Dasgupta-Heal economy?

Note: this is the **Rawlsian criterion**!

Step 1: for $c(t) = \bar{c}$, what is the efficient path for resource extraction $r(t)$?

$$\min_{r(\cdot)} \int_0^\infty r(t)dt \quad \text{with} \quad \dot{K}(t) = K(t)^\alpha r(t)^\beta - c(t), \;\; \dot{S}(t) = -r(t), \; c(t) = \bar{c}$$

Step 2: determine highest $\bar{c}$ for initial stocks $K(0)$, $S(0)$, and $r(t)$ from Step 1

Dasgupta-Heal-Solow

Result 1: the optimal extraction rule when $c(t) = \bar{c}$ is

$$r^*(\bar{c}, K(t)) = \left(\frac{\bar{c}}{1-\beta}\right)^{\frac{1}{\beta}} K(t)^{-\frac{\alpha}{\beta}}$$

and the Hotelling Rule, $\dot{F}_r / F_r = F_K$, holds.

Result 2: the maximum sustainable consumption is

$$\bar{c}^R = (1-\beta)(\alpha-\beta)^{\frac{\beta}{1-\beta}} S(0)^{\frac{\beta}{1-\beta}} K(0)^{\frac{\alpha-\beta}{1-\beta}}$$

Hartwick's Rule

Hartwick (1977): In the Dasgupta-Heal-Solow economy, along a costant consumption path that minimizes the cumulative resource extraction (i.e. the Hotelling rule holds), it must be that

$$\dot{K} - rF_r = 0$$

Interpretation: along the constant sustainable consumption path the total revenues from resource extraction, $p_r r$, where $p_r = F_r$, are fully reinvested in reproducible capital, $\dot{K}$.

The value of *net comprehensive investment* is zero! (note: the investment in the stock of resources $S(t)$ is negative, that is $-rF_r'$)

Hartwick's Rule: Proof

along the Solow solution

$$\bar{c}^R = (1 - \beta) K^\alpha r^\beta$$

investment is then

$$\dot{K} = K^\alpha r^\beta - c_0 = \beta K^\alpha r^\beta$$

now note that

$$\beta K^\alpha r^\beta = r \beta K^\alpha r^{\beta - 1} = r F_r'$$

hence

$$\dot{K} - r F_r = 0$$

Hartwick's Rule and Sustainability

Solow (1986): Hartwics' rule points to the general idea that maintaining the total productivity of capital stocks, understood in a comprehensive manner, so to include natural resources, biodversity, etc., allows a constant consumption to be sustained.

Comprehensive Net Investment: $I(t) \equiv \sum_{i=1}^{n} q_i(t)\dot{S}_i(t)$

Dixit-Hammond-Hoel (1980): along a competitive equilibrium path

$$\Delta(t)\frac{dU(c(t))}{dt} = \frac{dI(t)}{dt}$$

If net investment is zero, then utility under constant consumption is maximal.

Hartwick's Rule and Genuine Savings

Popular Intepretation of Hartwicks' rule: if society invests enough in human-made capital (technologies, infrastructures, knowledge, education), to compensate for the resource depletion and environmental degredation in order to maintain the value of comprehensive capital constant or non-decreasing, sustainability in the form of non-decreasing consumption is guaranteed.

Genuine Savings: $$\sum_{i=1}^{n} q_i(t)\dot{S}_i(t) \geq 0$$

Issues:

1. Hartwick's rule is neither sufficient not necessary for sustainability
2. Prices $q_i(t)$ must be those under an efficient *and* sustainable allocation

Sustainability and National Accounting

Optimal growth problem with mutliple capital stocks $(S_i, i = 1, ..., n)$

$$V(S(0)) = \max_{c(\cdot)} \int_0^\infty u(c(t), S(t)) e^{-\delta t} dt$$

$$\dot{S}_i(t) = r_i(S(t), c(t)), \quad S(0) \quad \text{given}$$

Hamiltonian: $$H(t) = u(c(t)) + \sum_{i=1}^{n} q_i(t) r_i(S(t), c(t))$$

Result (Asheim, 2007): General Case of Genuine Savings

$$\frac{dV}{dt} = \sum_{i=1}^{n} q_i(t) \dot{S}_i(t)$$

The change in welfare along the optimal path is equal to the change in the value of the capital stocks evaluated at the shadow prices.

Sustainability and National Accounting

Definition: An economy is sustainable at time t if $dV/dt \geq 0$.

Sustainability can be evaluated by measuring the change in the value of comprehensive capital, and requiring it to be non-negative

$$\sum_{i=1}^{n} q_i(t)\dot{S}_i(t) \geq 0$$

more on how this result is used in practice at the round table...

Summing Up

- we have studied the key elements in dynamic decision making: discounting, intergenerational inequality aversion, efficiency
- we have explored the implications of discounted utilitarianism for sustainability in models of nonrenewables, renewables, and reproducible capital
- we have introduced four alternative evaluation criteria to account for intergenerational equity, and explored their implications for sustainability
- in the process, we have learned about Hamiltonians, Optimal Control, Phase Diagrams, Steady States
- we briefly peaked into the implications of modeling sustainability for national accounting practices

take away: modeling sustainability requires an operational notion of intergenerational equity, one that is widely acceptable does not exist (yet)